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Reassessing the Ritz–Einstein debate on the radiation asymmetry in classical electrodynamics

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ABSTRACT

We investigate the debate between Walter Ritz and Albert Einstein on the origin and nature of the radiation asymmetry. We argue that Ritz's views on the radiation asymmetry were far richer and nuanced than the oft-cited joint letter with Einstein (Ritz & Einstein, 1909) suggests, and that Einstein's views in 1909 on the asymmetry are far more ambiguous than is commonly recognized. Indeed, there is strong evidence that Einstein ultimately came to agree with Ritz that elementary radiation processes in classical electrodynamics are non-symmetric and fully retarded.

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1. Introduction

Why do we observe radiation fields coherently diverging from a source but usually not fields coherently converging into a source? In a famous letter to the *Physikalische Zeitschrift*, Albert Einstein and Walter Ritz summarized their opposing views on the origin of the temporal arrow of radiation (Ritz & Einstein, 1909). While Ritz thought that the asymmetry is due to an asymmetry in the fundamental laws governing electromagnetic radiation, Einstein appears to have maintained that the irreversibility of radiation processes can be given a purely probabilistic explanation. This joint letter is frequently cited in philosophical discussions of the radiation asymmetry (see, e.g. Price, 1997; Zeh, 2007; Wheeler, Archibald, & Feynman, 1945; Norton, 2009; Earman, 2011) and almost always in order to appeal to Einstein's view in support of the idea that the radiation asymmetry ultimately is reducible to the very same statistical considerations that account for the thermodynamic asymmetry. The common view is that Einstein prevailed.

References to the Ritz–Einstein controversy usually do not go beyond a discussion of the joint letter. Yet once we consider further papers by Ritz and Einstein—Ritz, 1908a, 1908b, 1909, Einstein, 1909a preceding the joint letter, as well as a paper by Einstein (1909b) published later in the very same year also in the *Physikalische Zeitschrift*, shortly after Ritz's untimely death—a considerably more nuanced picture emerges. Ritz, whose own theory was an action-at-a-distance theory, offered several subtle criticisms of attempts to account for the asymmetry within a field-theoretic setting. Moreover, Einstein's last paper on the subject in that year raises a vexing interpretive puzzle concerning what Einstein's view on the asymmetry of radiation in classical electrodynamics were in 1909. One plausible reading of the exchange between Ritz and Einstein—and arguably a more plausible reading than the standard view—is that by the end of 1909 Ritz had convinced his former classmate Einstein that, within classical radiation theory, the irreversibility has its source at least partly in a fundamental asymmetry of elementary radiation processes.

In this paper we will trace the debate between Ritz and Einstein in more detail than is usually done in the literature. One of our aims is to set the historical record straight: Ritz's views on the arrow of radiation are far more interesting and nuanced than his casting in the role as Einstein's foil suggests; and Einstein's views are far too ambiguous for him to comfortably play the role of the 'hero' in defense of a purely statistical account of the radiation asymmetry. After a brief introduction to a contemporary understanding of what the temporal arrow of radiation consists in we will summarize the core arguments of Ritz's and Einstein's papers.

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We hope thereby to make these arguments, which originally appeared in German or French, available to a broader English speaking audience. Our interest, however, is not purely historical and it is our contention that contemporary discussions can benefit from a richer and less caricaturized understanding of the Ritz–Einstein debate.

2. The arrow of radiation

2.1. Converging vs. diverging waves and the representation argument

When electric charges accelerate, for example in an antenna, we observe a radiation field coherently diverging from the source. The time-reversed phenomenon—that is, radiation waves coherently converging into an accelerating source—is not something we observe. There are, as it is often put, coherently diverging but not coherently converging waves in nature. How can we explain this asymmetry? What might make this asymmetry appear to be especially puzzling and in need of an explanation is that the fundamental equations governing classical radiation phenomena, the Maxwell equations,[1] are time symmetric.

We said that the explanandum consists in the fact that there are coherently diverging but no coherently converging radiation fields in nature. Yet in one important sense this claim is false, since it is a mathematical fact that every radiation field can both be represented as involving diverging waves and as involving converging waves (*representation argument*). According to the modern understanding, classical electrodynamics is a field theory, with a dual ontology consisting of (ultimately microscopic) charged particles and electromagnetic fields. The temporal evolution of radiation fields associated with accelerating charged particles is governed by the inhomogeneous wave equation for the electric field

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\boldsymbol{E}=-4\pi\left(\nabla\rho+\frac{1}{c^2}\frac{\partial \boldsymbol{J}}{\partial t}\right) \quad (1)$$

and the magnetic field

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\boldsymbol{B}=\frac{4\pi}{c}\nabla\times\boldsymbol{J}, \quad (2)$$

which can be derived from the Maxwell equations. Commonly the wave equation is solved in terms of a modified initial value problem. The total field in a region of spacetime is given by the fields on an initial value surface together with the contribution of any field sources in that region, whose trajectories are taken to be given (and are not themselves determined as part of an initial value problem). In this case the contributions of the sources are diverging or so-called "retarded" fields.[2] That is, in an *initial* value problem the total field is represented as a combination of source-free incoming fields and retarded fields. But equally the total field can be represented in terms of a *final* value problem. In that case the contributions of the sources appear as converging or so-called "advanced" fields. One and the same total field F_{total}, thus, can be represented either as a combination of source-free incoming and retarded fields or as a combination of source-free outgoing and advanced fields:

$$F_{total}=F_{ret}+F_{in}=F_{adv}+F_{out} \quad (3)$$

What is more, the field can also be represented as a linear combination of retarded and advanced fields together with appropriate source-free fields.

For reasons of mathematical tractability the fields are in mathematical derivations usually replaced by the electromagnetic four-potential $A^\alpha=(\phi,\boldsymbol{A})$, which is determined by

$$\boldsymbol{E}=-\nabla\phi-\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t} \quad (4)$$

as well as

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}. \quad (5)$$

Since the potentials Φ and $\boldsymbol{A}$ are unique only up to a gauge transformation

$$(\phi',\boldsymbol{A}')=\left(\phi-\frac{1}{c}\frac{\partial\Lambda}{\partial t},\boldsymbol{A}+\nabla\Lambda\right) \quad (6)$$

with an arbitrary scalar function Λ, it is usually assumed in classical electrodynamics that the potentials merely serve a mathematical auxiliary function and, unlike the fields, do not correspond to anything physically real.

How a given total field is carved up into a component field associated with the sources present and a source-free field depends on the particular representation chosen: there is no unique way to carve up the total field. From a purely formal, mathematical standpoint, according to the so-called Kirchhoff representation theorem, which allows to derive the solution to the wave equation at an arbitrary point from the solution and its first-order derivative at all points on an arbitrary surface that encloses the point, neither a purely retarded nor a purely advanced field representation appears privileged. The difference in representations is solely due to whether we are choosing to represent the field in terms of an initial value problem or in terms of a final value problem (or an appropriate linear combination of the two). If we choose an initial value problem, then any fields at times before the sources 'turn on' appear as source-free fields and the sources formally contribute retarded fields after the sources turn on. If we choose a final value problem, then any fields at times after the sources 'turn off' appear as source-free fields and the sources formally contribute advanced fields before the sources turn off. Just as there is no unique field that is formally associated with the sources in a given problem, there is no unique source-free field: just as the question as to what component of the total field is mathematically associated with the field sources depends on our choice of initial or final value problem, so does the question as to what the source-free (or 'background') field is. There is no more the source-free field, independent of a particular choice of representation, as there is the field mathematically associated with a given configuration of sources. Without specifying a particular representation, the question as to whether sources are formally associated with retarded or advanced radiation has no answer, but—and this is important as well—once we are given the representation, there is nothing else we need to know in order to determine whether fields are retarded or advanced: if we represent the total field in terms of an initial value problem, then sources contribute retarded fields; and if we represent the field in terms of a final value problem, then sources contribute advanced radiation.

In what sense, then, is radiation asymmetric? The answer usually given today is that the asymmetry consists in the fact that the free incoming but not the free outgoing fields are approximately equal to zero. If incoming fields are equal to zero, then the total field can be represented as fully retarded field. Since outgoing fields will then generally be appreciably different from zero, the total field cannot be represented as being fully advanced. Thus, one common way to express the puzzle of the arrow of radiation is as follows: "Why does

[1] The Maxwell equations read in Gaussian units that are used throughout the manuscript: $\nabla\boldsymbol{E}=4\pi\rho$, $\nabla\times\boldsymbol{B}=\frac{4\pi}{c}\boldsymbol{J}+\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}$, $\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}$, $\nabla\boldsymbol{B}=0$, with the electric field $\boldsymbol{E}$, the magnetic field $\boldsymbol{B}$, the charge density ρ, the current density $\boldsymbol{J}$, and the velocity of light c.

[2] More exactly, retarded solutions calculate the potential or field by referring to a charge distribution in the past. Advanced solutions calculate the potential or field by referring to a charge distribution in the future. The identification of advanced with converging as well as of retarded with diverging can be justified on the basis of the Liénard–Wiechert potentials (12), which exhibit the corresponding properties.

the Sommerfeld radiation condition $F_{in}=0$ (in contrast to $F_{out}=0$) approximately apply in most situations?" (Zeh, 2007, p. 21) The Sommerfeld radiation condition is a temporal boundary condition—an initial value condition. Thus the asymmetry of the total radiation fields is expressed as an asymmetry concerning prevailing temporal boundary conditions and, hence, as an asymmetry between instantaneous states of the field: The fields on a spacelike hypersurface functioning as an initial value surface are approximately equal to zero, while the fields on a final value surface are generally not equal to zero.

This asymmetry is sometimes embedded in a cosmological context attempting to link the radiation asymmetry to the cosmological asymmetry of an expanding universe, essentially by drawing up a cosmological explanation for the claim that while radiation always originates in material sources, it is never completely absorbed. Arguably, these arguments remain inconclusive not least due to the considerable uncertainty concerning the material constitution of our universe, e.g. regarding the nature of the so-called dark energy and dark matter. In any case, they are currently not universally accepted.[3]

Here is how Sommerfeld himself characterizes the problem in the paper in which he first presented the exact boundary condition which ensures a unique solution to the wave equation and that the surface integral F_{in} vanishes in the limit, $\lim_{t\to-\infty}F_{in}=0$:

> "In optics and similar fields, one deals with *progressive waves* that radiate from the finite into infinity, i.e. *diverging waves*. Physically not realizable, but mathematically equivalent are waves that radiate from the infinite to be absorbed in certain source points in finite space-time, i.e. *converging waves*. By suitably combining both types of progressive waves, the sources can be totally eliminated and *standing waves* result of character of the eigen functions of the infinite domain. The possibility to superimpose such standing waves to every solution of the present problem shows the ambiguity of the problem. However, since nature of course realizes a uniquely determined solution of the problem, we conclude that an additional condition must be instantiated, which singles out progressive diverging waves from the manifold of solutions to the wave equation. The criterion will concern the behavior of waves at infinity; we will call it *radiation condition*."
>
> (Sommerfeld, 1968, p. 290; italics in the original).

That is, what Sommerfeld is looking for is a mathematical condition that can restrict the solution space of the equation to those solutions that are physically possible. Rather than taking the wave equation as delimiting the range of what is physically possible and then looking for an explanation of why a large class of physically possible solutions is not actualized, the problem for Sommerfeld seems to be with the mathematics: the wave equation has 'too many' solutions. The Sommerfeld radiation condition, according to this view, does not explain the asymmetry, but is merely the mathematical condition imposing a restriction on the electromagnetic field in large distances that enables us to exclude non-physical solutions of the wave equation and restrict the solutions to the physically plausible purely diverging waves.

2.2. *Retarded vs. advanced solutions and the action-at-a-distance view*

Some of the confusion in the debate on the radiation asymmetry stems from a lack of conceptual precision regarding the exact nature and origin of this asymmetry. Let us therefore distinguish three kinds of temporal asymmetries that will also be helpful in the later analysis of the Ritz–Einstein debate.

(1) The first asymmetry is the *converging–diverging asymmetry*. It consists in the observation that diverging waves are a ubiquitous phenomenon in nature, while converging waves are much rarer. As Einstein and others have argued, this asymmetry is a statistical asymmetry in that it can be explained in terms of the improbability of certain initial conditions. While initial conditions leading to converging waves are certainly possible, their occurrence presupposes highly unlikely correlations between spatially distant regions in the past. But, and this is a point we will return to below, in his last discussion of this issue in 1909 Einstein also emphasizes that the asymmetry is not purely statistical. For what, according to Einstein, is improbable are highly correlated arrangements of time-asymmetric elementary sources of radiation: "A spherical wave propagating inward is mathematically possible; but for its approximate realization an immense amount of emitting elementary structures are needed". (Einstein, 1909b, 821).

(2) Another asymmetry, the *retarded–advanced asymmetry*, amounts to the fact that in certain contexts the class of retarded solutions to the Maxwell equations is treated differently from the class of advanced solutions. When we are interested in modeling the field associated with a given charge or current configuration and the response of the distribution to that very field, assuming that the fields associated with the sources are advanced leads to results that are in contradiction with experience. One such example is discussed in Section 3.1.2: In the Lorentz–Abraham derivation of the radiation reaction, only the retarded solutions lead to the empirically correct force formula. In particular, advanced fields result in the wrong sign for the radiation reaction.
How does this square with the claim that the choice between retarded and advanced potentials is not a choice between different physical processes but only a matter of representation? If we assume that all (or at least all coherent) field disturbances are ultimately associated with field sources, then the formal symmetry in representations exists only if we assume that sources other than the charge distribution we are interested in modeling can vary freely to allow us to "make up" for putatively advanced fields associated with the distribution of interest to result in fields that look overall to be diverging. But this would require the existence of delicate initial correlations between the different sources. If we assume that such correlations do not exist, then only the use of retarded potentials gives empirically correct results.

(3) Finally, there is an asymmetry between emission and absorption processes. While retarded solutions to the Maxwell equations seem to account adequately for elementary emissions processes associated with individual charges, there do not appear to exist equivalent elementary absorption processes in a sense which we will discuss in more detail in the next section. The *emission–absorption asymmetry* constitutes the main reason why an oft-used argument does not go through in classical electrodynamics according to which retarded waves represent emission phenomena and advanced waves represent absorption phenomenona (*absorption argument*).
There is a related conceptual asymmetry in classical electrodynamics consisting in the fact that the Maxwell equations are used to determine the action of charged matter on the field, while the Lorentz force accounts for the action of the field on matter.

$$\boldsymbol{F}=\int \rho\boldsymbol{E}+\frac{1}{c}\boldsymbol{J}\times\boldsymbol{B}d^3x \tag{7}$$

[3] Connections between electrodynamics and cosmology are usually discussed within the Wheeler–Feynman framework of an action-at-a-distance classical electrodynamics (e.g. Hoyle & Narlikar, 1995; Zeh, 2007, Ch. 2.4; Frisch, 2005a, Ch. 6).

Note again that both the retarded–advanced asymmetry and the emission–absorption asymmetry are categorical asymmetries, i.e. they cannot be framed as statistical asymmetries in terms of the probability of boundary conditions. It therefore seems reasonable to restrict the solution space of Maxwell's equations to the retarded solutions. In Section 3.1.2 we will discuss various reasons that Ritz offers for why a restriction to retarded potentials is necessary in classical electrodynamics, evoking physical principles like the impossibility of a perpetuum mobile, Poynting's theorem for the conservation of energy in classical electrodynamics, and various derivations like that of radiation reaction. As already mentioned, Ritz implements the restriction by means of an action-at-a-distance theory.

The view that solutions to the Maxwell equations should be restricted to retarded potentials appears to have been widely accepted at the beginning of the 20th century. For instance, Hendrik Antoon Lorentz defends this perspective as well:

> "However, [the retarded potentials are] not the most general solution of the fundamental equations [. . .] and for example solutions are possible that show a propagation towards instead of from the volume elements. But of those we want to keep the theory free by assuming once and forever that the charged volume elements are really just starting points of disturbances of the equilibrium. We also exclude all states of the aether that do not depend on charged matter; if the latter were not there, the equilibrium of the aether would stay forever undisturbed." (Lorentz, 1904, 158–159, cited in Ritz, 1908a, 332; cf. also Lorentz, 1916, 240).[4]

Thus, similar to Sommerfeld Lorentz wanted to restrict solutions of the equations to those that treat charges as sources of radiation rather than sinks and want to exclude solutions to the source-free equations that involve combinations of retarded and advanced fields. It seems that Einstein in (1909b) eventually endorsed this viewpoint as well (see Section 3.6).

3. The Ritz–Einstein debate

3.1. Ritz's 'Recherches Critiques sur l'Électrodynamique Générale'

3.1.1. Ritz's action-at-a-distance approach to classical electrodynamics

The first paper we want to discuss is Ritz's momentous "Recherches Critiques sur l'Électrodynamique Générale" which appeared in *Annales de Chimie et de Physique* in February of 1908 (Ritz, 1908a). In this 130 pages long paper Ritz develops and defends a field-free action-at-a-distance theory[5] of electromagnetic interactions. The basic variables of Ritz's theory are only particle variables. He introduces an electromagnetic force-law (1908a, § II.2), which is similar though not fully identical to the action of one charged particle on another as determined by the retarded Liénard–Wiechert potentials $A^\alpha = (\Phi, \boldsymbol{A})$:

$$A^\alpha(\boldsymbol{x}, t) = \frac{1}{c}\int \frac{[J^\alpha(\boldsymbol{x}', t')]_{\text{ret}}}{R} d^3x' \tag{8}$$

with the four-vector current $J^\alpha = (c\rho, \boldsymbol{J})$ and $\boldsymbol{R} = \boldsymbol{x} - \boldsymbol{x}'$. $[. \ . \ .]_{\text{ret}}$ means that the quantity is evaluated at the time $t' = t - (R/c)$ for the retarded solutions.

Ritz also makes use of the fields, but these play only an auxiliary role in his theory as calculational devices and are not part of the theory's ontology. The time-asymmetric retarded potentials are fundamental, while the time-symmetric Maxwell equations positing electromagnetic fields have the status of mathematically auxiliary assumptions. Furthermore, since Ritz's framework posits direct particle–particle interactions, it also does not take the Lorentz force to be fundamental.

Thus, an implicit premise of both proponents of a field theoretic viewpoint and those of an action-at-a-distance approach is that there is a preferred formulation of the theory that also implies a fundamental ontology. Reformulations that rely on a different ontology are argued to be for various reasons only secondary or derived, while admittedly often useful in applications of the theory.

Part of Ritz's defense of his theory consists in a critique of the field theoretic framework and attempts to understand the radiation asymmetry within that context. Ritz is writing at a time, when an electromagnetic 'world picture' still appeared to be a possibility (cp. the account in Hon, 1995 of the Kaufmann experiments to determine if all mass is of electromagnetic nature). Thus, Ritz begins his paper by pointing out that "in recent years electric and electrodynamic phenomena have acquired an ever greater importance, encompassing optics, the laws of radiation as well as innumerable molecular phenomena" (317). Moreover, Lorentz's microscopic theory of the electron opened the prospect of a novel conception of nature in which the laws of electrodynamics are fundamental. (Ritz, 1908a, part I) offers a broad criticism of the electromagnetic project, arguing that the aether and field conceptions on which the Maxwell–Lorentz theory is based are deeply problematic. In particular, Ritz makes the following points:

1) Strictly formally, fields can be eliminated and can be replaced by direct inter-particle interactions (which, however, contrary to Newtonian gravitational interactions are not instantaneous), under the assumption that all fields are emitted and absorbed. This suggests an appeal to Ockham's razor and to ontological parsimony for eliminating field degrees of freedom.
2) The field equations have an infinity of solutions that do not represent observed phenomena. In particular, the equations permit a perpetuum mobile in the sense that they allow for an infinite amount of energy to be drawn from suitably arranged source-free fields. In order to restrict the solutions to what is observed, retarded potentials have to be introduced as an additional explicitly time-asymmetric assumption, since the time-asymmetry cannot, contrary to the received view, be derived from an asymmetry in initial conditions (see section 3.1.3).
3) There is no unique notion of the local energy of the field or aether. Ritz shows this in part I, §4 of his paper by suggesting an alternative expression for the energy content of the field that differs from Maxwell's formula $\frac{1}{8\pi}\int E^2 + B^2 d^3x$ in terms of energy distribution but is equally consistent with experience.
4) Gravitational forces cannot be reduced to electromagnetic interactions contrary to what an electromagnetic world-picture would have to assume.
5) Action and reaction are not equal in a theory that posits absolute velocities. Ritz calculates the force between two particles and shows that velocities and accelerations enter in asymmetric ways. He also notes that a theory presupposing an aether does not obey the equality of action and reaction, since the particle does not react back when the aether acts on a particle.
6) The experimental evidence at the time, especially the influential experiments by Kaufmann, does not compel us to conclude that the mass of the electron is of purely electromagnetic origin. This

[4] For a philosophical examination of some of Lorentz's views, see (Frisch, 2005b, 2011).

[5] There is some ambiguity how action at a distance should be understood. While Ritz and many other writers in the tradition presuppose that all fields are both emitted and absorbed, a weaker definition only posits that fields are secondary entities, the state of which can be fully derived from the arrangement of material particles. From this perspective, it is sufficient to assume that all fields originate in a material source but are not necessarily fully absorbed.

criticism aims at the project of an electromagnetic worldview and like item 4) must be understood within the historical context. With the subsequent development of physics, these issues become largely irrelevant.

7) Maxwell and Lorentz's theory presupposes an electromagnetic aether and thereby an absolute rest frame, which is incompatible with experience and needs to be replaced by a fully relative notion of space and time.

Ritz tries to address these issues within his own action-at-a-distance theory. Unlike other proponents of action at a distance, Ritz introduces fictive particles as mediators of electromagnetic interactions and thereby places himself in the tradition of a Newtonian emission theory of light: "fictive particles are emitted constantly in all direction from the electric charges; they continue to move in straight lines indefinitely and with a constant velocity, even through material objects" (321). It follows that the velocity of light depends on the source and is not constant, for which reason Ritz's theory has received some attention in anti-relativist circles. To the contemporary reader, however, the force and depth of Ritz's criticisms of the Lorentz–Maxwell theory are much more important than his own electromagnetic theory and the specific force-law that he develops in part II of his paper. While Ritz's critical comments are quite relevant to modern debates, his own theory is rooted in the context of the physics of his time and, in view of subsequent developments like the acceptance of relativity theory, seems outdated.[6]

3.1.2. The restriction to retarded potentials

After reviewing the Maxwell–Lorentz theory, including the introduction of retarded potentials as auxiliary device, Ritz presents his criticisms of the notions of an electric and magnetic field (Part I, §2). In the tradition of Kirchhoff and Mach, Ritz suggests that the notion of force even in classical mechanics is dispensable. Introducing the notion of force through appeals to our tactile experience of forces is scientifically problematic. Moreover, since mechanical forces are detectable only through observed displacements of material objects, the notion of force, so Ritz, need not be part of the fundamental physical principles. Thus it would be "regrettable" (329) if the notion of force was ineliminable in classical electrodynamics. Fortunately, he maintains, the notion can be eliminated, if we take retarded interactions between charged particles as fundamental.

Note that Ritz's criticism of the notion of force and his appeal to eliminate fields from classical electrodynamics are closely related. As it stands, the only role for fields is as carriers of forces. Consequently, without forces there is no need for fields. According to Ritz, this would be different, if there was evidence for an aether that is independent of charged matter, for example if the velocity of the aether had any physical significance. But he then refers to "interference experiments"—presumably including those of Michelson–Morley—to conclude that the aether thesis has been disproved (331).

For our purposes the core of Ritz's discussion consists of several arguments intended to show that one can posit retarded interactions as fundamental and derive from this the Maxwell–Lorentz equations as mathematically auxiliary equations, but that one cannot similarly begin with the Maxwell equations and arrive at the retarded potentials. The problem is that the Maxwell–Lorentz equations are time-symmetric, "while the two time directions play different roles in the retarded potentials and in the elementary actions" (Ritz, 1908a, 333).

[6] For a more in-depth analysis of Ritz's theory, see Martinez (2004) and O'Rahilly (1965, Ch. XI).

Ritz explains that the inhomogeneous wave equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)A^\alpha=\frac{4\pi}{c}J^\alpha, \tag{9}$$

here expressed in terms of the electromagnetic four-potential using the Lorenz gauge $\partial_\alpha A^\alpha = 0$, has different types of solution: (i) retarded solutions representing waves diverging from the source "which gave birth to them" (334); (ii) advanced solutions representing waves converging onto the source from past infinity or possibly from other sources; (iii) linear combination of the two solutions, centered on wave sources; (iv) solutions to the source free Maxwell equations that are combinations of converging and diverging waves and that hence may be centered on points in empty space. But according to Ritz, only the first type of solution represents phenomena we find in nature and hence we need an argument for rejecting the other kinds of solution.

Ritz gives a number of reasons for the restriction to retarded potentials. *First*, as long as there is no restriction to retarded potentials, centered on charged matter, the following is possible which according to Ritz amounts to a perpetuum mobile: An engine keeps moving indefinitely by drawing energy not from other matter but from the energy content of the aether or source-free fields (344). From this observation, Ritz draws a related criticism of Poyting's theorem as it is usually formulated in classical electrodynamics. Without restriction to retarded potentials, this theorem should not be considered as an electrodynamic version of energy conservation, exactly because it allows for a perpetuum mobile of the described kind. After all, the impossibility of a perpetuum mobile has traditionally been considered as one main motivation for energy conservation (ibd.).

Second, a number of derivations in classical electrodynamics require a non-statistical a priori restriction to retarded potentials: "one is led to reject [unphysical solutions such as advanced potentials] a priori, each time one calculates for example the electrical oscillations of a system (conducting sphere, Hertzian dipole, oscillating electron, etc.)" (334–335). Similar considerations appear to constitute the main motivation in many electrodynamics textbooks for rejecting advanced solutions usually by evoking some notion of causality. The brevity with which Ritz treats these issues which are crucial to his argument may indicate that this reasoning was widely accepted at the time.

Let us briefly analyze the status of the last argument with respect to contemporary classical electrodynamics. We will focus on derivations of radiation reaction and in particular on the following three derivations: (i) a derivation from Larmor's formula for the power radiated by an accelerated particle with charge q and acceleration a: $P=\frac{2}{3}\frac{q^2a^2}{c^3}$; (ii) the Lorentz–Abraham derivation calculating the self-interactions between the different parts of an extended accelerated particle; (iii) Dirac's derivation relying on renormalization.

All three derivations presuppose either a restriction to retarded potentials or at least ascribe asymmetric roles to retarded and advanced potentials. Regarding (i), if the Poynting vector $\boldsymbol{S}=\frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})$ is calculated using advanced instead of retarded fields, the power is radiated into the past and not the future. This changes the energy balance such that an accelerated particle undergoes an additional acceleration instead of radiation damping (cp. Jackson, 1999, 665). This is essentially Ritz's argument that unphysical solutions result from calculating the Poynting vector when using advanced potentials (344).

With respect to (iii), Dirac (1938) calculates the radiated fields as the difference between retarded and advanced field $F_{\rm rad}=F_{\rm out}-F_{\rm in}=F_{\rm ret}-F_{\rm adv}$, which introduces an asymmetric role for both types of fields. If radiation were emitted into the past instead of the future, we would have radiation acceleration instead of radiation damping: $F_{\rm rad}=F_{\rm in}-F_{\rm out}=F_{\rm adv}-F_{\rm ret}$.

Let us look in a bit more detail at the Abraham–Lorentz derivation (ii), which has been the most influential historically and is discussed in contemporary textbooks on classical electrodynamics (Jackson, 1999; Griffith, 2004). We will see that the derivation crucially relies on a restriction to retarded potentials. Our discussion follows Jackson (1999, Ch. 16.3, who in turn broadly follows Lorentz, 1916, note 18, p. 252) and we point out how the derivation would differ for advanced potentials. Jackson restricts his considerations to a particle instantaneously at rest with a charge distribution that is rigid and spherically symmetric. Under such circumstances the electromagnetic self-force is:

$$\left(\frac{dp}{dt}\right)_{em} = -\int \rho(\boldsymbol{x},t)\boldsymbol{E}_s(\boldsymbol{x},t)d^3x \tag{10}$$

where $\boldsymbol{E}_s$ is the electric self-field due to the charge distribution of the particle. Using the potentials $\boldsymbol{A}$ and Φ this can be expressed as:

$$\left(\frac{dp}{dt}\right)_{em} = \int \rho(\boldsymbol{x},t)\left[\nabla\Phi(\boldsymbol{x},t)+\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t}(\boldsymbol{x},t)\right]d^3x \tag{11}$$

The retarded Liénard–Wiechert potentials are used $A^\alpha=(\Phi,\boldsymbol{A})$, given by:

$$A^\alpha(\boldsymbol{x},t)=\frac{1}{c}\int\frac{[J^\alpha(\boldsymbol{x}',t)]_{ret/adv}}{R}d^3x' \tag{12}$$

with $J^\alpha=(c\rho,\boldsymbol{J})$ the four-vector current and $\boldsymbol{R}=\boldsymbol{x}-\boldsymbol{x}'$. $[\ldots]_{ret/adv}$ means that the quantity is evaluated at the time $t'=t-(R/c)$ for the retarded and $t'=t+(R/c)$ for the advanced solutions. Retarded quantities $[\ldots]_{ret}$ can be expressed in terms of the following Taylor expansion:

$$[\ldots]_{ret}=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{R}{c}\right)^n\frac{\partial^n}{\partial t^n}[\ldots]_{t'=t} \tag{13}$$

The corresponding formula for advanced quantities is the same without the factor $(-1)^n$, since the quantity is evaluated at $t'=t+(R/c)$. A somewhat lengthy calculation (see Jackson, 1999, Ch. 16.3) yields for the retarded expression:

$$\left(\frac{dp}{dt}\right)_{em} = \sum_{n=0}^{\infty}\frac{(-1)^n}{c^{n+2}}\frac{2}{3n!}\frac{\partial^{n+1}\boldsymbol{v}}{\partial t^{n+1}}\int d^3x'\int d^3x\rho(x')R^{n-1}\rho(\boldsymbol{x}) \tag{14}$$

Again, using advanced potentials, the result is the same without the factor $(-1)^n$. The term with $n=0$ is of order $\dot{\boldsymbol{v}}$ and therefore can be interpreted as an additional mass term. Note, however, that it diverges for point charges. It is the same for advanced and retarded potentials. Summands $n\geq 2$ are largely negligible in the limit of small particles. Finally, the summand with $n=1$ yields the well-known radiation reaction force:

$$\left(\frac{dp}{dt}\right)_{em} = -\frac{2}{3}\frac{e^2}{c^3}\ddot{\boldsymbol{v}} \tag{15}$$

which results in the equation of motion:

$$m\dot{\boldsymbol{v}}-\frac{2}{3}\frac{e^2}{c^3}\ddot{\boldsymbol{v}}=\boldsymbol{F}_{\mathbf{ext}} \tag{16}$$

Using advanced potentials, the sign in front of the radiation reaction force changes, which means that particles, whose acceleration increases would suffer an additional positive acceleration instead of radiation damping—just as for the other two derivations discussed above.[7] Therefore, using advanced potentials for the interactions in the particle would result in a formula contrary to experience.

A few comments are in order:

(1) One must be careful in stating the asymmetry required for the above derivation. The asymmetry consists in using retarded instead of advanced potentials for calculating the self-forces within the particle. In other words, the retarded-advanced asymmetry consists in using retarded potentials for calculating fields of charge elements, i.e. the asymmetry concerns elementary radiation processes.

(2) This asymmetry is compatible with the standard argument against an asymmetry between retarded and advanced fields which points out the equality of $F_{act}=F_{ret}+F_{in}=F_{adv}+F_{out}$. Of course, one can always construct fields that compensate any advanced fields in the required way, but this implies postulating complex source-free fields in addition to the advanced fields associated with the charged particle. In other words, the Abraham–Lorentz derivation starts with a straight-forward hypothesis what the relevant field-generating charge-distribution is, namely the extended charge density of the accelerated particle itself. Using retarded potentials, this hypothesis is corroborated in that the correct radiation-reaction force results. If we use advanced potentials, the observed damping force can only be derived by postulating an extremely complex free field F_{out}, compromising the overall simplicity of the argument.

(3) As has been noted for example by Einstein (1909b) and Popper (1956a, 1956b), the asymmetry between converging and diverging waves can be construed as a statistical asymmetry in that converging waves are possible but extremely improbable since they require a source of a large number of highly correlated particles. However, this argument for a statistical asymmetry between converging and diverging waves already presupposes the non-statistical asymmetry between retarded and advanced potentials, because the wave emitted by the highly correlated source particles is calculated using retarded fields or potentials. We thus are faced with both statistical and non-statistical asymmetries.

3.1.3. Implementing the restriction to retarded potentials

So far, we have seen important reasons for rejecting advanced solutions and for a restriction to retarded solutions. Hence, we need an argument how to implement the restriction. Ritz discusses several options. Some appeal to boundary conditions, others to a restriction in terms of laws. A standard proposal with respect to the former evokes the condition that at least at great distances the fields and their derivatives are equal to zero at some point in the past $t=0$, which is meant to restrict solutions to the purely retarded fields. But Ritz argues that this proposal is problematic for the following four reasons (335–336):

1) Ritz claims that the condition is hardly ever satisfied. If electrodynamic phenomena nevertheless exhibit the radiation asymmetry, then the condition cannot be necessary for the asymmetry to obtain.[8]

[7] Note that the additional acceleration occurs in positive time direction.

[8] Ritz also states that the condition is not satisfied in situations involving uniform translation or rotation. With this, Ritz may be aiming at relativity theories in particular of Lorentz and Einstein when he concludes that "the electromagnetic theory of uniform translation and rotation [. . .] remains thus excluded" (335). Still, the claim, which is not elaborated in further detail, is somewhat mysterious and we have not managed to determine its precise meaning. On one possible interpretation, Ritz is certainly right that uniformly moving charges always generate a field that is strictly speaking unequal to zero even at great distances. However, this obviously is also the case for charges at rest. Thus, the condition which Ritz formulates seems to require that at some moment in the past all matter was electrically neutral, which seems an overly strong assumption. At least partly this problem results from the fact that Ritz's condition is itself too demanding. It was Arnold Sommerfeld a few years later in Sommerfeld (1912) who published a precise mathematical requirement for the wave equation to have a unique solution u: $\lim_{r\to\infty} r\left(\frac{\partial u}{\partial r}-iku\right)=0$, where k is the wave number and i the imaginary unit.

2) If we merely demand that the incoming fields at $t=0$ are very weak, then this is compatible with there being converging fields of arbitrary strength at some later time. Therefore, the condition has to be that incoming fields (and their derivatives) are *strictly* zero at $t=0$, but this "kind of hypothesis is impermissible in physics" (335), Ritz claims. Thus, demanding merely approximately zero fields is not sufficient for the observed asymmetry. Alternatively one could prohibit converging waves by fiat and insist that any arbitrarily weak field cannot be converging, but this, Ritz claims would be question begging, in the context of the field theory. As Ritz points out, the problem in the case of electromagnetic waves is that they are not attenuated when they propagate in empty space. A similar problem does not arise for sound waves.
3) The presence of solar and stellar radiation requires that the time $t=0$ has to be placed beyond the limits of anything that is knowable. "But a hypothesis as fundamental [as the radiation condition] may not have such an impermissible character" (336).
4) If however we place $t=0$ at some finite time, then it follows that the fields *prior* to that moment are fully advanced, converging waves. Not only is this contrary to our experience, but it also, as we have seen, represents a type of perpetuum mobile. The charge continuously receives energy from the field converging from past infinity, without any other material body losing energy.

Ritz's criticisms are justified: there are indeed almost no circumstances in which incoming fields are strictly zero. But if we only impose the condition that incoming fields are approximately equal to zero, one cannot exclude the possibility that there are very small coherent fluctuations among the fields in distant regions, which result in a coherently converging field at later times. Moreover, imposing the field at any finite time $t=0$ has the problematic consequence that fields are fully advanced at times prior to that time. That is, under that assumption there is no global asymmetry, in the sense that the fully retarded fields for positive times will have their counterparts in the fully advanced fields for negative times. The only remaining option, which Ritz rejects on epistemological or methodological grounds, is to demand that appropriate initial conditions hold in the infinite past.

Ritz also considers a further attempt, due to Lorentz, to derive the asymmetry within the context of time-symmetric wave equations (338–339). Lorentz requires that a certain surface integral vanishes leading to a restriction to retarded fields. He explains that all field disturbances should be associated with charged particles and that the state of the aether is fully determined by the state of charged matter in that the aether remains "completely idle" when there are no charges present. In response Ritz points out that "there is no definite sense attached to the proposition: perturbations depending only to the aether are excluded" (338). Consider Eq. (3) above. Depending on the representation chosen (retarded or advanced or a linear combination of the two) different components of the total field will appear as free fields, "independent of the state of matter." If we adopt a retarded representation, then Lorentz's requirement suggests that we should set $F_{in}=0$. But if, by contrast, we adopt an advanced representation, then F_{out} should be equal to zero and, as Ritz argues, the total field would be equal to $F_{ret} + F_{in} = F_{adv}$. Since it is not the case in general that $F_{ret} = F_{adv}$ the two ways of implementing Lorentz's condition yield incompatible results. The problem with Lorentz's attempt to single out the retarded representation as privileged, Ritz emphasizes, is that "*the decomposition of a wave-field is a mathematical operation that can be done in an infinite number of different ways.*" (339, italics in the original).

Thus, Ritz chooses to implement the restriction to retarded potentials on the level of physical laws. He concludes from his discussion that the only manner of accounting for the asymmetry of radiation is by adopting *a priori*, as he says, the retarded potentials, "which distinguish elementary actions" (339). Thus, "*it is the formula of the elementary actions, and not the system of equations involving partial derivatives, which is the exact and complete expression of Lorentz's theory* [. . .] [These equations] *and the notion of the ether are fundamentally incapable of expressing the set of laws of the propagation of electromagnetic actions*." (339, italics in the original).

3.2. Ritz's "Über die Grundlagen der Elektrodynamik und die Theorie der Schwarzen Strahlung"

The main topic of Ritz's (1908b) paper in *Physikalische Zeitschrift*, which constitutes the opening move in the Ritz–Einstein debate, is the problem of black-body radiation. According to a classical field-theoretic treatment of black-body radiation—leading to the Rayleigh–Jeans formula for the spectral density $u(f,T)=\frac{8\pi f^2}{c^3}k_BT$, which holds in the limit of a large number of photons $hf \ll k_BT$—the power radiated by a black body diverges, as higher and higher frequencies of the radiation field are included. The physicist Paul Ehrenfest called this problem the "ultraviolet catastrophe". Ritz argues that the problem is a consequence of treating the aether (or electromagnetic field) as possessing independent degrees of freedom. Ritz suggests that the problem can be avoided in a retarded action-at-a-distance theory that only has particle degrees of freedom, which are finite. In particular, the diverging modes of the cavity radiation in the field theory cannot be understood as the retarded fields associated with charges in the reflecting cavity walls. In the derivation it is assumed that the walls are perfectly reflecting, but this presupposes an infinite number of charges and precisely this idealization is "impermissible" in the present context (see 497). From a modern point of view, field quantization is the key to solving the problem of the ultraviolet catastrophe as is shown for example in Einstein (1917)—a solution that Ritz explicitly and in hindsight wrongly rejects.

In order to motivate his particle theory, Ritz repeats the criticisms made in (1908a) of the Maxwell–Lorentz field theory and the condition that the fields be zero at some time t_0: the Maxwell–Lorentz equations have not only retarded solutions, but also advanced solutions and linear combinations of the two. As in the earlier paper he writes these two solutions as follows:

$$f_1(x,y,z,t)=\frac{1}{4\pi}\int\frac{\phi(x',y',z',t-\frac{r}{c})}{r}dx'dy'dz' \quad (17)$$

$$f_2(x,y,z,t)=\frac{1}{4\pi}\int\frac{\phi(x',y',z',t+\frac{r}{c})}{r}dx'dy'dz' \quad (18)$$

Here c is the speed of light, ϕ is the charge configuration, and both f_1 and f_2 are assumed to vanish at infinity. The retarded solution f_1 specifies the potential at t as a function of the state of the sources at the earlier time $t-\frac{r}{c}$, while the advanced solution f_2 specifies the potential in terms of the state of the sources at the later time $t+\frac{r}{c}$.

Ritz's first criticism is that fully advanced solutions are unphysical since they represent a physical object that receives energy from the infinite without any other object losing any amount of energy. "Such an object, which would be capable of continuously receiving energy from the aether in this manner, would have to be called a *perpetuum mobile* and is physically impossible"(495). Ritz then repeats his criticism of the condition that the field is zero at some time t_0. It is obvious from Ritz's discussion of this condition (and this will become important further in Ritz's disagreement with Einstein) that what Ritz means by the integrals f_1 and f_2 for the potentials are expressions for the *total* potential or field: f_1 results, or so it is claimed, when we demand that the fields (and their derivatives) are zero at some initial time t_0. That is, the hope of defenders of the radiation condition is that this condition can ensure that the field in the future of t_0 is approximately fully retarded and that this is both necessary and sufficient to capture the sense in which radiation phenomena are irreversible. But, Ritz argues, demanding that the field is zero at some time t_0, is problematic for several now familiar reasons: the condition prohibits many physically possible situations such as uniform translation; it implies that the fields prior to t_0 are fully advanced; and if the condition only holds approximately ("what alone can be asserted" (496)), then converging fields are not excluded, since, for a hyperbolic equation such as the wave equation, there could be very weak convergent fields at t_0 that become arbitrarily strong at some later time.

Thus, Ritz asserts that "the complete expression of the laws of radiation and of Maxwell's theory in general does not consist in the differential equations but in the elementary actions, which arise from the introduction of the retarded potentials into Lorentz's expression of the ponderomotive force [i.e. the Lorentz force]" (496). As in the earlier paper, Ritz concludes that once we eliminate free-field solutions and independent degrees of freedom of the aether from our theory, the aether becomes "a pure abstraction" and, in accord with our experiences, "completely banned from physics" (502). But, Ritz continues, "thereby disappears one of the main foundations of the Maxwellian description of the phenomena through partial differential equations, which no longer have any physical meaning but only have the status of mathematical intermediary constructs" (502). Ritz thus joins in the critique of the aether that was quite prevalent at the time. Yet while Einstein criticizes the aether from within a field-theoretic approach, Ritz does so from an action-at-a-distance viewpoint.

3.3. *Einstein's "Zum Gegenwärtigen Stand des Strahlungsproblems"*

Einstein's first contribution to the debate with Ritz is an answer to Ritz's (1908b) and to papers by H. A. Lorentz and J. H. Jeans on the problem of blackbody radiation. We will here focus on the first section of Einstein's paper, which contains Einstein's reply to Ritz.

In accord with the accepted view, Einstein appears to endorse the opposite explanatory relation between the Maxwell equations and the retarded potentials. While for Ritz the latter are primary, Einstein maintains that the retarded potentials are "only mathematical auxiliary forms." But curiously he also says the following, echoing Ritz's term of an "intermediary construct" ("mathematische Zwischenkonstruktion"): "It is surely correct that the Maxwell equations for empty space, considered on their own, say nothing ["sagen gar nichts aus"], that they are only intermediary constructs; the same can, as is well known, be said of Newton's equations of motion or any other theory that needs to be supplemented by other theories to deliver a representation of a complex of phenomena" (Einstein, 1909a, 185). Einstein's concession here to Ritz is puzzling. Newton's equations of motion do not say anything about the phenomena in the sense that they need to be supplemented by a specific force law. Similarly, one might say that the source-free Maxwell equations say nothing about the motion of charged objects, unless we are also told how electromagnetic fields couple to sources and are given the Lorentz force law. But if we restrict our attention to regions of empty space, then the source-free Maxwell equations do allow us to set up an initial value problem for a source-free volume. They determine the state of the field completely, given the field on an appropriate boundary surface, and it is unclear what any other theory could contribute to the representation of the state of the field in those regions. The source-free Maxwell equations might not tell us everything, but they also do not tell us nothing. This contrasts sharply with the case of Newton's theory, which cannot represent the motion of any object, unless supplemented by a concrete force law.

The Maxwell equations for empty space are straightforwardly intermediary constructs on an action-at-a-distance interpretation that denies the reality of electromagnetic fields, but Einstein offers two closely related arguments against such an interpretation—independently of the issue of time-reversibility (for a discussion cf. also Pietsch, 2010). *First,* in Ritz's retarded action-at-a-distance theory the "energy principle"—the principle of energy conservation—does not hold locally. This is so because the energy radiated away by an accelerated charge is not balanced locally by an increase in the energy in the field and at best shows up at some later time as the energy increase of another charged particle with which the radiating charge interacts. *Second* and relatedly, in a retarded action-at-a-distance theory the instantaneous state of the system does not suffice to determine the system's time-evolution. A light pulse emitted by a source, Einstein points out, is not represented in the system at times between the emission event and when the light pulse is received at a screen.

These two criticisms are surely correct. If we demand that energy conservation holds locally and that our theories satisfy the continuous-time Markov property and represent the evolution of a system as depending only on the instantaneous state of a system, then a retarded action-at-a-distance theory has to be rejected. Note however that the field theoretic picture of energy conservation is not as unproblematic as it might seem. As emphasized by Ritz, the energy distribution in the field is underdetermined in classical electrodynamics (1908a, part I, §4). Ritz argues that there exist "an infinity" of possible alternatives to the common Maxwellian expression of the field energy

$$\frac{1}{8\pi}\int E^2 + B^2 d^3x. \tag{19}$$

The latter is only particularly simple (341). Related arguments can still be found in contemporary textbooks. For example, Griffiths argues that an electrodynamics based on potentials employs a notion of energy completely distinct from an electrodynamics based on fields (2004, §2.4.4). One could say that the former uses a relational, the latter a substantival energy concept, which imply different viewpoints on how energy is distributed. In the end, this issue appears to be only resolved by quantum mechanics, which employs an energy concept that is fundamentally incompatible with the Maxwellian—at least on the microscopic level. While in quantum electrodynamics, energy is thought to be localized in photons, classical electrodynamics considers energy to be spread out in space.

Einstein also criticizes Ritz's discussion of the role of the different solutions to the wave equation. Ritz, as we have seen, claims that (17), (18), and linear combinations of the two are different solutions to the wave equations and that the field theory has no satisfactory account of restricting these solutions to the retarded solution (17). Einstein argues that this involves an elementary error: the two integrals written down by Ritz, (17) and (18), are not different solutions representing different field configurations but rather constitute different representations of one and the same field—a line of reasoning that we had termed 'representation argument' in Section 2.1. In the retarded representation, the field is represented as depending on the state of the sources at earlier times, while in the advanced representation the field is represented as depending on the state of the sources at later times. The total field in both cases is one and the same—only the representation of the field is different. As Einstein puts it, "in the first case we calculate the electromagnetic field from the totality of the processes that create it, in the second case we calculate the field from the totality of absorption processes" (186).

But Einstein's argument is wrong (*pace* Earman, who cites it approvingly[9]) at least from the standpoint of a particle-field ontology. As we have seen in Section 2, Einstein is correct in that the total field can be given a retarded or an advanced representation. But in general these representations will involve source free fields in addition to the fields associated with sources. While according to (3) every field can be represented equivalently as sum of retarded and incoming source-free fields or as sum of advanced and source-free outgoing fields, it is not the case in general that $F_{ret} = F_{adv}$. But as we have seen above, f_1 and f_2, the two fields written down by Ritz, are the purely retarded and advanced fields, respectively, and these will in general not be equal, given the knowledge of all relevant charge distributions. Now, Einstein's claim that the field can equivalently be represented by the totality of the emission or absorption processes suggests that he assumes that all emitted radiation is eventually absorbed. Indeed, he maintains that *both* the assumption of retarded radiation that is emitted into future infinity and is never absorbed and the assumption of source-free radiation coming in from past infinity involve illegitimate and paradoxical invocations of the infinite. But this ‘full absorption assumption’ is a substantial and controversial additional assumption that does not follow from the field-theoretic framework alone. Even then Einstein's further claim that any radiation processes *in a strictly finite space* can equivalently be represented as fully retarded or as fully advanced may not be correct. The only reading under which the claim is true (given the full absorption assumption) is that there will be some finite but perhaps very large volume such that the total field in that volume can be represented as fully advanced.

If every field could be represented as both fully retarded and fully advanced, it becomes puzzling why we take radiation fields to exhibit a characteristic asymmetry. Why does it seem to us that there are diverging but no coherently converging fields in nature? Einstein hints at a statistical explanation ending his discussion of Ritz's view with the following intriguing remark: “Moreover we cannot conclude from the fact that [pure absorption] processes are not observable that electromagnetic elementary processes are irreversible, just as we cannot conclude that the elementary motions of atoms are irreversible from the second law of thermodynamics.” (Einstein, 1909a, 186) The rest of the paper is a discussion of Planck's radiation formula including comments how the classical theory should be changed to account for the formula, mainly by introducing field quantization and quantization of absorption and emission processes.

3.4. Ritz's “Zum Gegenwärtigen Stand des Strahlungsproblems. (Erwiderung auf den Aufsatz von Herrn A. Einstein.)”

In his reply to Einstein, Ritz insists that the fully retarded and the fully advanced solutions to the wave equations do indeed represent different physical processes, rather than being different representations of one and the same total field. In general, Ritz insists, the fully retarded and the fully advanced fields associated with a source are not equal: “A retarded and advanced process cannot be made to coincide simply by reversing the sign of the time [that is, replacing t with $-t$]. Thus, we are here not faced with a different kind of calculation but with a different process.” (Ritz, 1909, 224) Ritz goes on to point out what in his view is Einstein's mistake: a general solution to the field equations contains a surface integral that is independent of the state of the sources, our F_{in} and F_{out} above. F_{in} is a solution to the homogenous Maxwell equations, which by the so-called Kirchhoff representation theorem can be shown to be equal to a surface integral over the past spatial and temporal boundaries. The standard explanation of the radiation asymmetry argues that the surface integral is zero in the retarded representation, but this implies that F_{out} will in general not be equal to zero: “But the Lorentzian assumptions consists in the claim that when we use f_1 and presuppose a large space, then the surface integral vanishes, from which it follows that, if instead we use f_2 for the same process, the surface integral will in general not vanish” (224).

In reply to Einstein's objection concerning local energy conservation, Ritz argues that to the extent that solutions to the Maxwell equations represent physically observable processes, what we can derive from the instantaneous state of the field will agree with what can be derived from the integral over the retarded sources. To the extent, then, that the two formulations are observationally equivalent, the field representation cannot be superior. But the field representation also has unphysical solutions, which can be excluded only by assuming retarded interactions. Ritz concludes that until the asymmetry can be derived successfully with the help of suitable auxiliary assumptions within the field-theoretic framework, he “will view the fact that the retarded forces are the only true integrals of these equations (into cold outer space), and that in great distances energy always flows outward or at least never inward, as the root of irreversibility and of the Second Law [of thermodynamics]” (Ritz, 1909, 225).

In summary, while Einstein insists that advanced solutions constitute a different representation of electromagnetic fields, Ritz claims that the advanced solutions calculated in classical electrodynamics do not constitute processes that are observed in physics. As we will see, Einstein eventually conceded to Ritz that according to the usual formulation of classical electrodynamics the advanced solutions do not adequately account for absorption processes, and he came to believe that this is a fundamental problem of the theory.

3.5. Ritz and Einstein's “Zum Gegenwärtigen Stand des Strahlungsproblems.”

Ritz and Einstein's famous joint letter (1909) constitutes the final episode in their debate concerning the arrow of radiation. The letter's explicit aim is “to clear up the disagreement in opinion” (323) between Ritz and Einstein, but in the end they agree to disagree, trying to make explicit the presuppositions of their disagreeing views. There is much about the short letter that is deeply confusing and, perhaps, deeply confused.

The letter states that “in the special cases in which an electric and magnetic process remains restricted to a finite space, the process can be represented in the form of [the integral (17)] as well as in the form of [the integral (18)] as well as in other forms.” This, of course, was Einstein's claim in (Einstein, 1909a), a claim that Ritz had correctly pointed out is false since it ignores the surface integrals, which will in general not all be zero. In particular, if a purely retarded representation (17) is adequate, the advanced representation will, in addition to (18) in general include a free field term that is independent of the sources. Moreover, in his criticism of the standard constraint on initial conditions, Ritz had also argued that a purely retarded representation of the *fields* without a contribution of incoming fields is not general enough and cannot adequately represent many phenomena, other than a restriction to a representation in terms of retarded *potentials*. Thus, why Ritz might have agreed with Einstein's claim that in finite volumes the total field can be given a purely retarded representation, as well as a purely advanced representation, is puzzling.

Einstein, the letter says, thought that it was possible to restrict oneself to considering finite spaces without restricting the

[9] “But Einstein (1909a) claimed that the representation by means of retarded potentials is not more special than the representation by, say, a linear combination of retarded and advanced potentials, both being representations of the same solution.” (Earman, 2011, 490)

generality of the discussion, i.e. he seems to assume a condition that all radiation is eventually absorbed, whereas Ritz takes this restriction as in principle impermissible. The letter continues with the following oft-quoted conclusion:

> If one adopts this [Ritz's] standpoint, then experience compels us to consider the representation by means of retarded potentials as the only one possible, if one is inclined to the view that the fact of the irreversibility of radiation must already find its expression in the fundamental equations. Ritz considers the restriction to the form of retarded potentials as one of the roots of the second law [of thermodynamics], while Einstein believes that the irreversibility is exclusively due to reasons of probability (Ritz & Einstein, 1909, 324).

The view that the retarded potentials are the correct ones to use is doubly hedged: not only does this conclusion presuppose that we consider radiation into the infinite, but also that we are antecedently committed to locating the asymmetry in the fundamental laws. There is no reference to any of Ritz's arguments for the latter assumption and against attempts to derive the asymmetry from a special initial condition.

The final sentence of the letter picks up apparently opposing suggestions by Ritz and Einstein in their earlier papers: Ritz's suggestion that the restriction of retarded potential and, more generally, the assumption that ‘energy flows only outward’ is at the root of the second law; and Einstein's suggestion that the irreversibility of radiation processes, like the irreversibility of thermodynamic processes, ultimately has a probabilistic explanation.

3.6. Einstein's “Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung” and “Zur Quantentheorie der Strahlung”

Three months after the joint letter had been submitted to the *Physikalische Zeitschrift* Ritz died at age 31, having succumbed to his many-year-long fight with tuberculosis. Later that year Einstein held a talk on “the nature and constitution of radiation” which was published in *Physikalische Zeitschrift* in October of 1909. In this talk Einstein first retraces some of the developments that led to the theory of relativity and the rejection of the aether hypothesis and then examines reasons for abandoning a purely classical conception of radiation and replacing it with a quantum hypothesis.

In the talk Einstein does not mention Ritz or the exchange between them, but two passages in the paper are rather remarkable in light of Einstein's criticism of Ritz's retarded emission theory of radiation. *First*, Einstein says that there are phenomena that indicate that “light has certain fundamental properties, which are more readily understood from the standpoint of the Newtonian emission theory of light than from the standpoint of the wave theory. Therefore I am of the opinion that the next phase in the development of theoretical physics will result in a theory of light that can be understood as a fusion of the wave and emission theories of light” (817). While this points in the direction of an emission theory of the type Ritz had supported, an important difference is that Einstein eventually adopts directed emission while Ritz had assumed isotropic emission.

Second, and more important for our purposes here, he says the following about the classical wave theory of light:

> The basic property of the wave theory, which results in these problems, seems to me to be the following. While in kinetic molecular theory there exists an inverse process for every process, in which only a small number of elementary particles participate, for example for every molecular collision, this is not the case for elementary radiation processes, according to the wave theory. According to the theory familiar to us, an oscillating ion produces a spherical wave that propagates outward. The inverse process does not exist as *elementary process*. A spherical wave propagating inward is mathematically possible; but for its approximate realization an immense amount of emitting elementary structures are needed. Elementary processes of the emission of light as such are, thus, not reversible. Here, I believe, the wave theory is incorrect (Einstein, 1909b, 821; his italics).

According to Einstein, the retarded solutions adequately account for elementary emission processes without the need to postulate a vast number of absorbing charges, while advanced solutions only describe elementary absorption processes in the practically impossible case of an infinite number of highly correlated emitting charges. Thus, Einstein here explicitly asserts what he earlier in the very same year appears to have denied, namely that elementary radiation processes are time-asymmetric—in contrast to elementary processes in molecular mechanics. Thus, there is a clear disanalogy between the thermodynamic asymmetry and the radiation asymmetry. What is more, Einstein claims that this asymmetry exists for the wave theory of radiation, whereas Ritz had argued that positing asymmetric elementary actions was an argument in favor of an action-at-a-distance theory.

To be sure, Einstein takes the irreversibility of elementary radiation processes in the classical wave theory to be problematic. His main reason for this is that the energy of the wave is dispersed as the wave spreads from the source, which is in tension with experimental evidence that suggests that the entire emitted energy ought to be available for elementary absorption processes, as manifested for example in the photo effect treated by Einstein in 1905. With respect to such phenomena, Einstein says “Newton's emission theory of light seems to contain more truth” (821). Nevertheless, he claims unequivocally that the classical wave theory of radiation posits irreversible elementary emission processes.

It is difficult to render Einstein's discussion here consistent with his earlier claims that the irreversibility is “exclusively due to reason of probability” and that we cannot conclude that “electromagnetic elementary processes are irreversible, just as we cannot conclude that the elementary motions of atoms are irreversible from the second law of thermodynamics”. One might try to argue that when Einstein says that the irreversibility is due to reasons of probability, he means that this will turn out to be the correct explanation in whatever theory ultimately proves to be adequate and that this is compatible with holding that the wave theory posits asymmetric elementary processes. But the focus of the joint letter clearly is classical radiation theory and, hence, this attempt to construe Einstein's view in a consistent manner appears somewhat strained. A more plausible interpretation is that in the end Ritz did manage to convince Einstein that in the classical theory elementary radiation processes have to be understood as irreversible. In his brief discussion of the Ritz–Einstein debate Earman says that “the predominate opinion had been that Einstein prevailed” (2011, 486). At least as Einstein himself was concerned, this assessment may be wrong and it may be that Einstein came to agree with Ritz on the source of the irreversibility in the classical theory.

In the end, it seems to have been a crucial step for Einstein to recognize the asymmetry in classical electrodynamics, which permitted him to determine the ways in which the classical theory goes wrong. It eventually led to his important contributions to quantum electrodynamics including his 1916 derivation of Planck's formula in “On the quantum theory of radiation”. The crucial assumptions were:

1) Classical electrodynamics does not yield the correct picture concerning elementary radiation processes in that it assumes that energy is dispersed in spherical waves. Rather, “the

elementary process of radiation emission seems to be directed" (821). Einstein specifies that a consistent theory can only be derived by assuming *fully* directed processes.

2) Relatedly, the distribution of energy and momentum in the field is not continuous, but rather is quantized with photons as elements of quantization. Naturally, emission and absorption processes are also quantized.
3) To every emission process there exists a corresponding absorption process. It is this assumption, which allows Einstein in his 1916 paper to formulate a condition of equilibrium between particles and fields.

Presumably, the reversibility of electrodynamics can be recovered under these additional assumptions since the asymmetry between emission and absorption processes is removed. Consequently, it becomes possible to interpret advanced solutions as absorption phenomena as postulated by the absorption argument of Section 2.2. The radial nature of macroscopic emission processes would turn out a statistical phenomenon. Consequently, the macroscopic radiation field must be considered a statistical entity as well.

However, all this is only possible by moving far beyond the framework of classical electrodynamics and the implications for classical electrodynamics have never been coherently worked out. Thus, the non-statistical retarded-advanced asymmetry concerning elementary radiation processes remains a feature of classical electrodynamics and the Einstein–Ritz debate has never really been brought to a satisfying conclusion.

4. Conclusion

While Ritz and Einstein's joint letter is widely cited in discussions of the radiation asymmetry (Price, 1997; Zeh, 2007; Wheeler et al., 1945; North, 2003; Earman, 2011), the papers by Ritz and Einstein preceding the letter receive almost no attention and Einstein's 1909b is completely ignored. One notable exception in the latter respect is a letter by Karl Popper to the journal *Nature* (Popper, 1956b). In an earlier letter Popper argued that the process of waves spreading on a surface of water after a stone is dropped, exhibits an irreversibility that is distinct from the thermodynamic asymmetry. The reverse process of circularly converging waves, according to Popper, "cannot be regarded as a possible classical process." He went on to say that "[the reverse process] would demand a vast number of distant coherent generators of waves the coordination of which, to be explicable, would have to be shown, in [a film depicting the process], as originating from the center. This however, raises precisely the same difficulty again, if we try to reverse the amended film" (Popper, 1956a, 538). Popper's claim that a coherently converging wave would require a vast number of coherent generators is reminiscent of Einstein's claim that for a collapsing wave to be approximately realized "an immense amount of emitting elementary structures are needed." Popper himself noted the similarity in a second letter to *Nature*: "I have found since that nearly half a century ago, Einstein used a somewhat similar argument. Had I known this, I would not have written my communication" (Popper, 1956b). But while Popper's analysis seems to be correct that the asymmetry of radiation is somewhat distinct from the thermodynamical asymmetry, he does not discuss it in the context of classical electrodynamics.

As we have seen the exact nature of the *non-statistical* asymmetry in classical electrodynamics can be stated more precisely: a restriction to retarded potentials or fields is required when considering *elementary* radiation processes. In particular, in certain situations with a clear intuition what the relevant charges are, correct predictions only follow from taking charge elements as sources of retarded potentials and not as sinks of advanced potentials, e.g. when calculating the radiation reaction from the retarded self-forces. Note again that this asymmetry of elementary radiation processes is compatible with the representation argument concerning the equivalence (3) of incoming plus retarded with outgoing plus advanced fields. Yet this conclusion is incompatible with the standard view, often (and as it seems wrongly) attributed to Einstein, that the radiation asymmetry is just another instance of a statistical asymmetry in analogy to the thermodynamical asymmetry.

References

Dirac Paul, A. M. (1938). Classical theory of radiating electrons.. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, *167*(929), 148–169.

Earman, John (2011). Sharpening the electromagnetic arrow(s) of time. In: Craig Callender (Ed.), *The Oxford Handbook of Philosophy of Time* (pp. 485–528). Oxford: Oxford University Press.

Einstein, Albert (1909a). Über die Entwicklung unserer Anschauung über das Wesen und die Konstitution der Strahlung. *Physikalische Zeitschrift*, *10*(22), 817–825.

Einstein, Albert (1909b). Zum gegenwärtigen Stand des Strahlungsproblems. *Physikalische Zeitschrift*, *10*(6), 185–193.

Einstein, Albert (1917). Zur Quantentheorie der Strahlung. *Physikalische Zeitschrift*, *18*, 121–128.

Frisch, Mathias (2005a). *Inconsistency, asymmetry, and non-locality: a philosophical investigation of classical electrodynamics*. Oxford: Oxford University Press.

Frisch, Mathias (2005b). Mechanisms, principles, and Lorentz's cautious realism. *Studies in History and Philosophy of Science Part B*, *36*(4), 659–679.

Frisch, Mathias (2011b). Principle or constructive relativity. *Studies in History and Philosophy of Science Part B*, *42*(3), 176–183.

Griffiths, David J. (2004). *Introduction to electrodynamics*. New Delhi: Prentice-Hall of India.

Hon, Giora (1995). Is the identification of experimental error contextually dependent? The case of Kaufmann's experiment and its varied reception In: Jed Z. Buchwald (Ed.), *Scientific Practice: Theories and Stories of Doing Physics* (pp. 170–223). Chicago, IL: The University of Chicago Press.

Hoyle, Fred, & Narlikar, Jayant Vishnu (1995). Cosmology and action-at-a-distance electrodynamics. *Reviews of Modern Physics*, *67*(1), 113–155.

Jackson, John David (1999). *Classical electrodynamics*. New York: Wiley.

Lorentz, Hendrik Antoon (1904). Weiterbildung der Maxwellschen Theorie. Elektronentheorie. In *Encyclopädie der mathematischen Wissenschaften*: *Vol. 5*, Nr. 2. pp. 145–288.

Lorentz, Hendrik Antoon (1916). *The theory of electrons*. B.G. Teubner: Leipzig & Berlin.

Martinez, Alberto (2004). Ritz, Einstein, and the emission hypothesis. *Physics in Perspective*, *6*(1), 4–28.

North, Jill (2003). Understanding the time-asymmetry of radiation. *Philosophy of Science*, *70*, 1086–1097.

Norton, John (2009). Is there an independent principle of causality in physics? *British Journal for the Philosophy of Science*, *60*(3), 475–486.

O'Rahilly, Alfred (1965). *Electromagnetic theory*. New York: Dover.

Pietsch, Wolfgang (2010). On conceptual problems in classical electrodynamics: prospects and problems of an action-at-a-distance interpretation. *Studies in History and Philosophy of Modern Physics*, *41*, 67–77.

Popper, K. R. (1956a). The arrow of time. *Nature*, *177*(4507)), http://dx.doi.org/10.1038/177538a0.

Popper, K. R. (1956b). Irreversibility and mechanics. *Nature*, *178*(4529)), http://dx.doi.org/10.1038/178382a0.

Price, Huw (1997). *Time's arrow & Archimedes' point : new directions for the physics of time*. New York: Oxford University Press.

Ritz, Walther (1908a). Recherches critiques sur l'électrodynamique générale. *Annales Délelőtt Chimie et Délelőtt Physique*, *13*, 145–275 Reprinted in Ritz (1911), 317-426.

Ritz, Walther (1908b). Über die Grundlagen der Elektrodynamik und die Theorie der schwarzen Strahlung. *Physikalische Zeitschrift* (pp. 493–502), 493–502.

Ritz, Walther (1909). Zum gegenwärtigen Stand des Strahlungsproblems. *Physikalische Zeitschrift*, *10*(7), 224–225 Reprinted in Ritz (1911), 503-506.

Ritz, Walther (1911). *Gesammelte werke – oeuvres. herausgegeben von societe suisse de physique*. Paris: Gauthier-Villars.

Ritz, Walther, & Einstein, Albert (1909). Zum gegenwärtigen Stand des Strahlungsproblems. *Physikalische Zeitschrift*, *10*(9), 323–324.

Sommerfeld, Arnold (1912). Die Greensche Funktion der Schwingungsgleichung. *Jahresberichte des Deutschen Mathematischen Vereins*, *21*, 309–353.

Sommerfeld, Arnold (1968). *Gesammelte Werke, Bd. 1*. Braunschweig: Friedrich Vieweg & Sohn.

Wheeler, John, Archibald, & Feynman, Richard Phillips (1945). Interaction with the absorber as the mechanism of radiation. *Reviews of Modern Physics*, *17*(2-3), 157–181. http://dx.doi.org/10.1103/RevModPhys.17.157.

Zeh, H. D. (2007). *The physical basis of the direction of time*. Berlin; New York: Springer. ⟨http://public.eblib.com/EBLPublic/PublicView.do?ptiID=372947⟩.