

Analysis of modern observations of the Sun and inner planets

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Abstract. This paper presents an analysis of the world-wide optical observations of the Sun, Mercury and Venus obtained over the time span 1960–1990 with 17 meridian instruments and 2 astrolabes and published by 1993. A procedure of reduction of planetary observations from one to another reference frame is proposed and applied to the initial observational material. The observations have been transformed to the FK5 system and compared with the ephemeris DE200. A method of intercomparison of individual instrumental series for determination of systematic “goodness” of an instrument has been developed. Weights then have been assigned to each instrumental series composing the material under analysis. These have then been used for construction of the mean instrumental systems and applied in the LS solutions. Spectral analysis of ($O - C$) residuals has revealed the existence of strong sidereal harmonics in declination for Mercury and Venus, giving evidence of errors in the comparison ephemeris. An apparent positive trend in right ascension residuals for the Sun and both planets is detected. Conditional equations, used for LS solutions to determine corrections to orbital parameters of the planets and orientation parameters of the FK5, are presented; their external accuracy is tested by use of simulated data consisting of a series of differences in apparent positions from DE119 and Newcomb’s theory at different time intervals. It is shown that for the Sun, Mercury and Venus over the selected 30-year interval, conditional equations give consistent results within $0.04''$ for constant terms and within $0.2'' \text{ cy}^{-1}$ for secular variations, while those obtained for Mars are in great discrepancy.

The results of the weighted LS solutions on the basis of the observational material appear to be consistent within $0.1''$ for the Sun and both planets. The mean value of the equator correction to the FK5 derived from Mercury and Venus observations is $0.062'' \pm 0.023''$, while the observations of the Sun give $-0.002'' \pm 0.009''$. The mean value of the equinox correction to the FK5 from the Sun, Mercury and Venus observations at the epoch 1973.0 is $0.103''$. An anomalous negative secular variation of the equinox correction has been detected for the Sun and Venus, estimated to be $(-1.216 \pm 0.274)'' \text{ cy}^{-1}$, close in absolute value to the non-precessional equinox motion introduced into FK4 proper motions when constructing the FK5. A positive drift in right ascension residuals for Mercury was found

to be $(0.722 \pm 0.268)'' \text{ cy}^{-1}$. The results on frames orientation are compared with those by other authors and discussed.

Key words: astrometry – reference systems – ephemerides

1. Introduction

Orientation of the fundamental system FK5 to the dynamical reference frame of Newcomb’s planetary theory is based on the study of Fricke (1982). In that work, the equinox correction to the FK4 is derived from a certain selection of determinations made by different authors on the basis of series of observations of the Sun and major planets, photographic observations of minor planets obtained at different observatories from 1900 to 1977, and lunar occultations resulting from observations over a 150-year interval of time. In most cases of individual results, each determination is based upon relatively short series of 10–15 years resulting from observation of a certain object with a certain instrument. Sometimes the reported corrections represent mean values determined from several instrumental series, as Fricke’s correction for Mercury and Venus. Only in a few cases (Duma et al. 1980; Glebova 1975) and, in recent years, (Nimi 1990) were the equator and equinox corrections determined as a combined LS solution using observations from different instruments. In these studies, as usual, individual instrumental systems were not analysed, and observations from different series were included in the solution with equal weights.

Assembled together, individual equator and equinox corrections obtained in this way during the 20th century give an impressive image of uncertainty in estimation of these fundamental astronomical constants, as is seen from the comprehensive analysis of Sveshnikov (1985). Dispersion of the results attains $\pm 0.4''$ for equator and $\pm 0.6''$ for equinox corrections. In many cases, corrections derived by one author from observations with a certain instrument for different objects are in discrepancy on the order 0.1–0.3''. It is clear that the main cause of these discrepancies originates in instrumental systematic errors of observations and, sometimes, in the differences in reduction methods.

This gives evidence of a certain methodological shortcoming of the approach in which the orientation parameters of the

Table 1. Statistical summary for instrumental series of the Sun observations in right ascension and declination; K – ordinal number of a series in the PLANETS database; NOBS – number of observations; YEARS – interval of observations; P_i – internal weight of the instrument; P_e – external weight of the instrument. At the bottom are the total number of observations and the weighted means of P_e

INSTRUMENT	K	RIGHT ASCENSION				DECLINATION			
		NOBS	YEARS	P_i	P_e	NOBS	YEARS	P_i	P_e
NIK TR.I	01	2246	61.1- 89.9	1.02	0.89				
NIK V.C	02					2201	60.0- 85.9	0.82	0.89
NIK TR.C	03	98	61.7- 63.5	0.45	0.89	83	61.7- 63.5	0.66	0.60
TASH TR.C	04	1323	60.7- 89.4	1.00	0.83				
PULK TR.I	05	905	60.1- 76.5	0.57	0.91				
PULK V.C	06					754	60.1- 76.8	1.01	0.79
PULK TR.C	07	47	61.1- 61.8	1.07	0.77				
GOL V.C	08					726	71.2- 87.1	1.10	0.93
MOSC TR.C	09	168	61.8- 75.9	0.73	0.66				
WASH TR.C	14	1390	60.0- 76.3	2.26	0.96	1345	60.0- 76.3	1.33	0.95
GREEN TR.C	15	1059	60.0- 82.3	2.67	0.87	988	60.0- 82.3	1.38	0.95
BELGR TR.C	18	442	74.8- 88.7	2.17	0.77	525	74.9- 88.7	4.72	0.88
KISL V.C	19					471	84.4- 88.0	2.64	0.98
KISL TR.I	20	42	81.7- 82.7	1.38	0.24				
CERGA AST	21	687	76.4- 89.0	2.28	0.92				
SIM AST	22	69	87.5- 91.6	2.32	0.93				
		8481			0.88	7093			0.87

fundamental system are determined as a mean value of some selected individual corrections. A basically new approach in determination of relative orientation of dynamical and stellar frames, realized by Standish (1982), could essentially improve the systematic quality of a result provided that a maximum possible number of optical observational series, properly analysed and reduced to a common reference system, were used in the creation of the ephemerides.

Meanwhile, the observational material accumulated by this time with all world-wide astrometric instruments represents a large and poorly used resource of astrometry. This holds true especially for modern observations of the Sun and major planets, which have been an important part of observational activity for most of astrometric instruments during last 3 decades. Only a relatively small fraction of them has actually been used for improvement of ephemerides and orientation of the fundamental catalogues. This situation is caused, in our opinion, by the following factors. There is still not any centre in the astronomical community, similar to CDS, engaged in collecting and distributing planetary observational data in machine-readable form. Majority of optical planetary observations are dispersed in numerous editions, some of which are often inaccessible to all astronomers; some series of observations are unpublished, being in possession of an institute or an observer. Creation of such a data base including more-or-less complete set of the world-wide series of planetary observations by one person or even by a group of astronomers is a rather difficult task. As a consequence, there is still not any mathematical technique dealing with com-

parative analysis of planetary observational series which would allow to assess their systematic goodness in an external sense to be considered with this type of observations. The only exceptions known to us are the studies of Kharin (1984;1986). That is why, in most cases, only a priori “good” observations, such as USNO, Greenwich, Cape series are used as basic material in different studies, while other series are often not taken into account as “less good”, without any justification.

In this study, we have made an attempt to overcome the methodological restrictions outlined above. We have tried to perform a global analysis of a maximum available to us optical observations of the Sun and inner planets obtained with meridian instruments and astrolabes over the period from 1960 to 1990. We have proposed a method of intercomparison of observations made with different instruments to estimate the systematic quality of instrumental series incorporated in the analysis. We have thus defined a mean instrumental system for each object and equatorial coordinate via a set of weights determined as a result of intercomparison. The weighted instrumental series then have been used in LS solutions for equator and equinox corrections.

The observational material collected and used is presented in Sect. 2. The reduction procedure to a common stellar system, a system of constants and an ephemeris of comparison is described in Sect. 3. Section 4 describes the method and the results of intercomparison of instrumental series that have been used to construct the weighting system of the actual configuration of instruments. The residuals ($O - C$) in right ascension and declination are analysed in Sect. 5. The conditional equa-

Table 2. Statistical summary for instrumental series of Mercury observations in right ascension and declination; K – ordinal number of a series in the PLANETS database; NOBS – number of observations; YEARS – interval of observations; P_i – internal weight of the instrument; P_e – external weight of the instrument. At the bottom are the total number of observations and the weighted means of P_e

INSTRUMENT	K	RIGHT ASCENSION				DECLINATION			
		NOBS	YEARS	P_i	P_e	NOBS	YEARS	P_i	P_e
NIK TR.I	01	491	61.1- 89.8	1.32	0.68				
NIK V.C	02					314	60.1- 85.8	1.22	0.60
NIK TR.C	03	29	61.7- 63.5	0.63	0.79	32	62.3- 63.5	0.79	0.55
TASH TR.C	04	208	62.0- 89.2	2.48	0.45				
PULK TR.I	05	121	60.7- 76.6	1.03	0.49				
PULK V.C	06					133	61.3- 76.8	2.56	0.34
PULK TR.C	07	11	61.3- 61.9	1.22	0.53				
GOL V.C	08					56	71.5- 87.1	2.82	0.35
MOSC TR.C	09	42	62.3- 75.9	0.92	0.61				
KAZ TR.C	11					13	72.7- 75.4	0.67	0.36
WASH TR.C	14	431	60.0- 77.5	1.75	0.76	437	60.0- 77.5	1.22	0.75
GREEN TR.C	15	286	60.1- 82.3	3.66	0.58	276	60.1- 82.3	2.13	0.55
TOKYO TR.C	16	54	64.1- 77.4	1.25	0.44	56	64.1- 77.3	1.28	0.44
BELGR TR.C	18	134	74.9- 88.6	2.26	0.60	146	74.9- 88.6	7.20	0.49
KISL V.C	19					199	84.1- 87.9	4.22	0.96
KISL TR.I	20	29	81.7- 82.7	3.49	0.68				
		1838			0.63	1653			0.63

tions chosen for analysis in this study are presented and tested by use of simulated data in Sect. 6. And, finally, the results of LS solutions on orientation parameters of the FK5 are presented and discussed in Sect. 7.

2. Material

To perform a global analysis of the maximum available worldwide instrumental series, according to our initial objective, we have limited ourselves in this first study with only modern observations. Our reasons were follows: a) observational activity has reached maximum intensity during last 3 decades, so that, provided this material having been collected, it would become possible to realize a classical astrometric scheme forming some “mean instrumental system” on the basis of multiple series of observations to minimize individual systematic errors of each series in a joint solution; b) modern observations represent a higher qualitative level of instrumentation and astrometric technique in methods of observations and their primary reduction compared with older ones and, therefore, they might be considered as systematically more reliable; c) many of series obtained in recent decades have not contributed to the improvement of the FK5; d) and, at last, we had to limit ourselves with a certain time span, to perform this research in a reasonable term.

The material used in this study represent published (and some unpublished) observations [in the form of the differences ($O - C$)] of the Sun, Mercury and Venus obtained in the period 1960–1990 with transit circles (TR.C), transit instruments

(TR.I), vertical circles (V.C) and astrolabes (AST) in the following observatories: Nikolaev (NIK), Tashkent (TASH), Pulkovo (PULK), Goloseevo (GOL), Moscow (MOSC), Kazan (KAZ), Kharkov (KHAR), Kislovodsk (KISL), Simeis (SIM), Belgrade (BELGR), Washington (WASH), Greenwich (GREEN), Tokyo, Grasse (CERGA). None observational series has been excluded for the reasons of low accuracy of the results. If some series appear to be absent, that is because they have been unavailable to us or results have been unpublished by the time of preparation of this paper. The total number of observations and time spans of the series are listed in Tables 1–3. Multiple daily observations of right ascensions with the CERGA solar astrolabe were averaged and taken as one correction for a certain date. More comprehensive description of the instruments, methods of observation and corresponding references will be given elsewhere in a separate paper.

The actual temporal distribution of observations made with each instrument for each object and equatorial coordinate is presented in Fig. 1a–f. These figures give a good idea of the activity in observation of the Sun and inner planets with almost all worldwide transit circles and astrolabes during the recent 30 years. The first impression is a discontinuity and a non-uniformity of the observations. Different series cover different time intervals, have different temporal density and, sometimes, have lacunae of 5–10 years. Some of the instruments, which have produced a rather long series, such as the GREEN TR.C, PULK TR.I, PULK V.C, GOL V.C, have already finished their observational activity; others (BELGR TR.C, KISL V.C, KISL TR.I, CERGA

Table 3. Statistical summary for instrumental series of Venus observations in right ascension and declination; K – ordinal number of a series in the PLANETS database; NOBS – number of observations; YEARS – interval of observations; P_i – internal weight of the instrument; P_e – external weight of the instrument. At the bottom are the total number of observations and the weighted means of P_e

INSTRUMENT	K	RIGHT ASCENSION				DECLINATION			
		NOBS	YEARS	P_i	P_e	NOBS	YEARS	P_i	P_e
NIK TR.I	01	1883	61.1- 89.9	1.32	0.82				
NIK V.C	02					1682	60.0- 85.9	0.99	0.87
NIK TR.C	03	84	61.7- 63.5	0.59	0.90	94	61.7- 63.5	0.83	0.80
TASH TR.C	04	966	60.7- 89.4	1.45	0.50				
PULK TR.I	05	602	60.6- 76.6	0.81	0.82				
PULK V.C	06					508	61.1- 76.6	1.50	0.61
PULK TR.C	07	48	61.3- 61.9	1.07	0.89				
GOL V.C	08					586	69.4- 87.1	1.59	0.74
MOSC TR.C	09	128	61.8- 75.8	1.39	0.56				
KHAR TR.C	10	55	64.8- 84.6	4.80	0.16				
KAZ TR.C	11					152	61.2- 76.7	0.97	0.66
WASH TR.C	14	1084	60.0- 77.5	1.71	0.86	1055	60.0- 77.5	1.07	0.83
GREEN TR.C	15	677	60.1- 82.3	4.24	0.56	631	60.1- 82.3	2.15	0.79
TOKYO TR.C	16	116	64.1- 77.2	1.70	0.46	115	64.1- 77.3	0.94	0.56
BELGR TR.C	18	416	74.9- 88.7	2.30	0.55	437	74.9- 88.7	4.15	0.62
KISL V.C	19					329	84.3- 88.0	3.13	0.94
KISL TR.I	20	47	81.7- 82.7	1.92	0.66				
		6106			0.71	5589			0.79

AST) have only begun observations in the past 20 years. There are also instruments, such as KISL TR.I, NIK TR.C, PULK TR.C, KAZ TR.C, which still obtained quite a small number of observations. It is seen also that the number of meridian instruments which continue to produce astrometric data on the Sun and inner planets has fallen sharply during recent years; WASH TR.C, NIK V.C, NIK TR.I, TASH TR.C, BELGR TR.C, KISL V.C, KISL TR.I along with the USNO 9-inch TR.C at Black Birch and Tokyo PMC. Observations of the latter instruments have still not been published. But the best existing astrometric instruments, those at La Palma and Bordeaux, do not participate in this kind of observations. Augmented activity of astrolabes in observations of the Sun during recent years is, however, reassuring. We note that aside from the work of the CERGA solar astrolabe and the Sao Paolo astrolabe, observations of the Sun commenced or planned in Simeis (Kolesnik 1987), Paris (Chollet & Noël 1990), Cagliari (Poma & Zanzu 1990), San Fernando (Sánchez et al. 1993), Santiago de Chile (Noël 1993) and Malatya (Turkey).

3. Reduction of the observations to the FK5 system and to the ephemeris DE200

Obviously, the most correct way to obtain the results of differential observations in some reference system is to re-reduce them in this system. This assumes an availability of original obser-

vational material and is possible only within an institute where observations have been made. In practice, only in very few cases after publication, are observations re-reduced in a new reference system when the astronomical constants and fundamental catalogue are changed. Since analysis of old observations is now becoming the most progressive way to improve modern ephemerides and astronomical constants, the matter of practical interest is a possibility of the more-or-less correct reduction of observations from one reference system to another without complete re-reduction of the original observational material.

All published values collected for this research ($O - C$) $_{\alpha}$ and ($O - C$) $_{\delta}$ represent differences between stellar (α_0, δ_0) and ephemeris (α_c, δ_c) apparent positions of an object (Sun or a planet) at the time of observation t . Originally, these positions were based on various catalogue systems (individual absolute catalogues, such as USNO's W4/50 and W5/50, or fundamental catalogues, FK3, FK4, FK5), which are referred to different standard epochs and equinoxes, B1950 or J2000. Apparent places of reference stars and planets were computed using two systems of astronomical constants (IAU 1964 and IAU 1976, 1980). Newcomb's theory, the DE200 and the VSOP82 served as ephemerides of comparison. Apparent places of planets were calculated sometimes with either preliminary values of differences between Ephemeris Time and Universal Time or even without this correction. To reduce the observations to the FK5-based stellar system and the DE200 as an ephemeris of comparison,

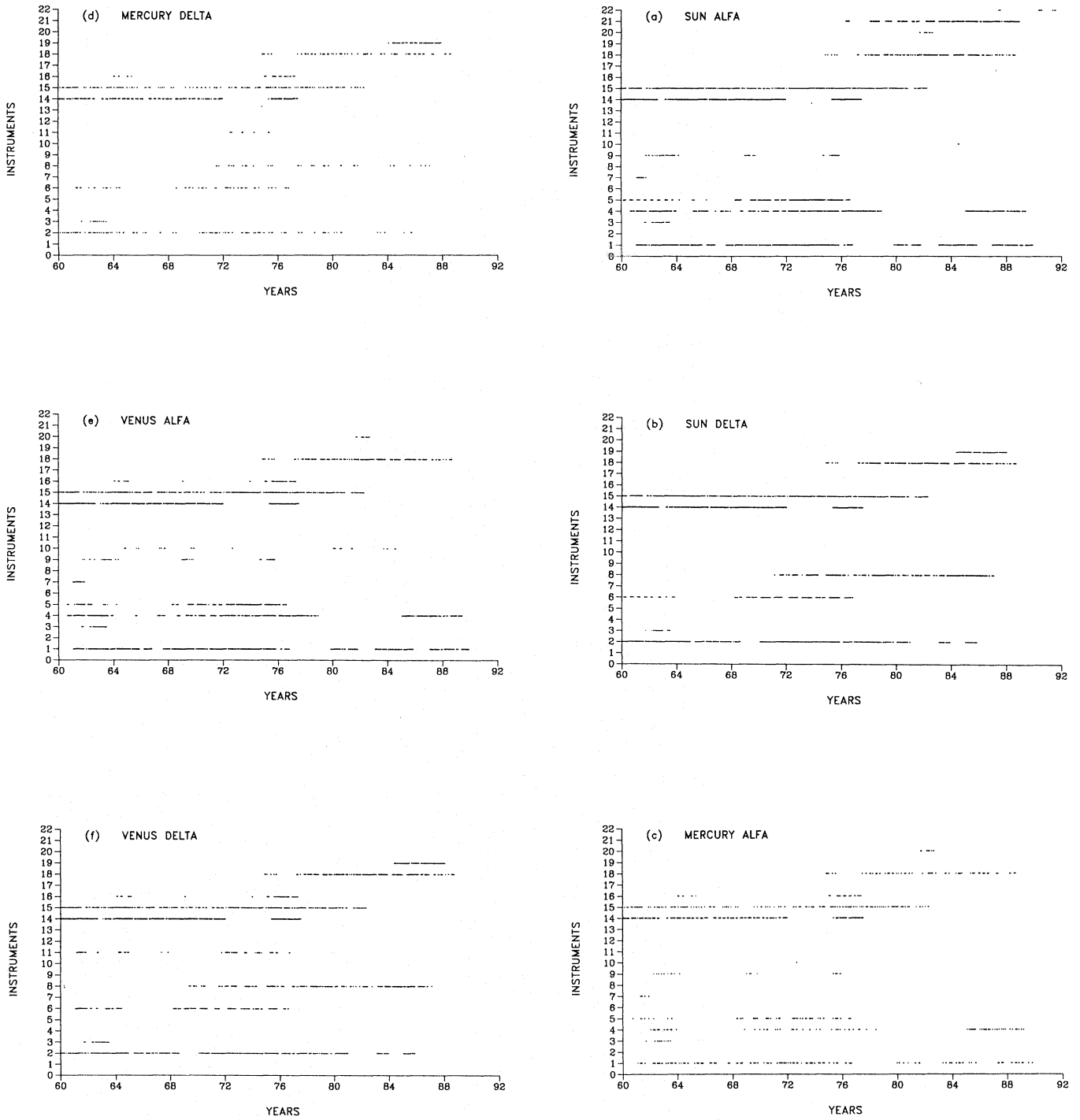


Fig. 1a-f. Temporal distribution of observations in the instrumental series used in this study in right ascension and declination respectively for the Sun (a and b), Mercury (c and d), and Venus (e and f). The series are represented by the set of dots, each corresponding to one observation, against their ordinal number in the PLANETS database, indicated on abscissa. The ordinal numbers of the series are presented in the Tables 1-3

the following corrections have been applied to the observational material presented in the previous section:

(a) *Correction for the “stellar” apparent place of an object*

Stellar positions of an object are derived with respect to an equator point and a value of clock correction computed from apparent places of a certain set of reference stars observed with this object. The rigorous correction for transformation from one reference system to another will be hence the difference between values of instrumental constants defining orientation of an instrument in space calculated in each system from the same reference stars, associated with observation of the object. One can compute these differences only when having a list of those reference stars. If not, it is possible to reproduce only a systematic distortion of a stellar reference system in the point of the sky where an object was observed at the epoch t , which is caused by changing of a reference catalogue and a procedure of apparent places computation. One could estimate this distortion by forming residuals between apparent places computed in both systems of some fictitious star, the position of which coincide with the object’s position at the time of observation. As applied to FK4 and FK5-based systems, this approximate procedure would be like follows:

$$\begin{aligned} \begin{bmatrix} \Delta\alpha_0 \\ \Delta\delta_0 \end{bmatrix}_t &= f_{(J2000 \Rightarrow t)} \left\{ h_{(B1950 \Rightarrow J2000)} [(\alpha_{B1950} + \Delta\alpha), \right. \\ &\quad \left. (\delta_{B1950} + \Delta\delta), \Delta\mu, \Delta\mu', \pi = 0, \nu = 0 \right\} \\ &- g_{(B1950 \Rightarrow t)} \left\{ \alpha_{B1950}, \delta_{B1950}, \right. \\ &\quad \left. \mu = 0, \mu' = 0, \pi = 0, \nu = 0 \right\} \end{aligned} \quad (1)$$

where α_{B1950} and δ_{B1950} are fictitious equatorial positions resulting from precessional transformation of the object’s mean position α_t and δ_t , at the epoch of transit to the epoch B1950 with the standards of IAU 1964; f and g are symbolic notations of the standard procedures of apparent places computation according to the standards IAU 1976, 1980 (Lederle & Schwan 1984) and IAU 1964 (Explanatory Supplement to the A.E. 1968) respectively; h is the conventional procedure in the IAU 1976 system for transforming mean places from B1950 to J2000 (Aoki et al. 1983); π and ν are the parallax and radial velocity used in these procedures and both equal to zero in this case, as unknowns; $\Delta\alpha$, $\Delta\delta$, $\Delta\mu$, $\Delta\mu'$ are positional and proper motion systematic differences between FK5 and the catalogue to which the observations are referred. If the reference catalogue is the FK4, they are calculated from the tabular values given in (Fricke et al. 1988). If FK3 or some catalogue in instrumental system, they represent the sums of (FK5–FK4) differences and the corresponding differences (FK4–Cat). In the case of the so-called “quasi-FK5 system” in which the Apparent Places of Fundamental Stars were published from 1984 to 1987, formula (1) degenerates into simple tabular (FK5–FK4) corrections:

$$\begin{bmatrix} \Delta\alpha_0 \\ \Delta\delta_0 \end{bmatrix}_t = \begin{bmatrix} \Delta\alpha + \Delta\mu(t - 1950) \\ \Delta\delta + \Delta\mu'(t - 1950) \end{bmatrix}.$$

To neglect the anomalous gravitational deflection of light in the case when fictitious positions are of the Sun, the corrections (1) should be taken as mean values for some four points surrounding the Sun at the epoch t at distances not less than 10° .

In this study, we have used observations of the Sun made with two astrolabes, the Simeis classical astrolabe and the CERGA solar astrolabe. Observations of the Sun with the equal altitude method are not strictly differential, since reference stars are inaccessible for an astrolabe during daytime. Observations, therefore, are referred via a plumb line of the instrument to an instantaneous pole. The orientation of the latter is determined either from smoothed values of time and latitude corrections, derived from night time observations of fundamental groups of stars, or from Earth orientation parameters published by BIH, now IERS, using some adopted value of the mean longitude and latitude of the instrument. The first method was applied for primary reduction of observations in CERGA from 1978 to 1980 (Chollet 1981) and in Simeis (Kolesnik 1987). The second has been used for CERGA observations in 1976, and from 1981 onwards. According to (Débarbat & Guinot 1970), the influence of astronomical constants on determination of time and latitude corrections with astrolabes may be estimated as the difference of apparent places calculated in both systems of some “zenith star” with right ascension equal to the mean sidereal time of a group observation. Thus, in the case of CERGA observations in 1978, for reduction from the FK4 to the FK5 system, we have used (1) with a fictitious right ascension equal to the local sidereal time at midnight, declination equal to the latitude of the instrument, and $\Delta\alpha$, $\Delta\delta$, $\Delta\mu$, $\Delta\mu'$ equal to zero, since astrolabe observations are quasi-absolute. CERGA observations from 1979 to 1988 have been re-reduced in the FK5-based system and kindly sent to us by the observers (Journet 1991).

(b) *Correction for the “ephemeris” apparent place of an object*

$$\begin{bmatrix} \Delta\alpha_c \\ \Delta\delta_c \end{bmatrix}_t = \begin{bmatrix} \alpha_{DE200} - \alpha_N \\ \delta_{DE200} - \delta_N \end{bmatrix}_t \quad (2)$$

where α_{DE200} , δ_{DE200} and α_N , δ_N are apparent equatorial coordinates of an object computed at the time of its transit on the basis of the ephemeris DE200 with the standards IAU 1976, 1980 (Kaplan et al. 1989), and from Newcomb’s planetary theory with the standards IAU 1964 (Explanatory Supplement to the A.E. 1968) respectively. Positions given by the theory VSOP82 on our level of analysis was considered to be identical with those from DE200.

(c) *Correction for inaccurate value of difference between Ephemeris time and Universal time*

$$\begin{bmatrix} \Delta\alpha_T \\ \Delta\delta_T \end{bmatrix}_t = \begin{bmatrix} \dot{\alpha} (\Delta T - \Delta T_p) \\ \dot{\delta} (\Delta T - \Delta T_p) \end{bmatrix}_t \quad (3)$$

where ΔT_p is a preliminary value of the difference ($ET - UT$) used by authors of publications for computation of ephemeris apparent places of an object, ΔT is its final value taken from

Astronomical Almanac 1992 and interpolated at the epoch of transit, $\dot{\alpha}$, $\dot{\delta}$ are the planet's equatorial coordinate rates at the moment of observation.

Thus, original residuals, $(O - C)^{\text{OLD}}$ referred to a system different from FK5 and compared with Newcomb's theory, have been used to form $(O - C)^{\text{NEW}}$, referred to the FK5 system and DE200, by applying corrections (1), (2), (3).

$$\begin{bmatrix} (O - C)_\alpha \\ (O - C)_\delta \end{bmatrix}_t^{\text{NEW}} = \begin{bmatrix} (O - C)_\alpha \\ (O - C)_\delta \end{bmatrix}_t^{\text{OLD}} + \begin{bmatrix} \Delta\alpha_0 \\ \Delta\delta_0 \end{bmatrix}_t - \begin{bmatrix} \Delta\alpha_c \\ \Delta\delta_c \end{bmatrix}_t - \begin{bmatrix} \Delta\alpha_T \\ \Delta\delta_T \end{bmatrix}_t.$$

Since an apparent correlation of the Sun's $(O - C)_\delta$ residuals with the zenith distance of observation has been detected for almost all instrumental series, an additional correction of the type $\Delta\delta_z = A + Btgz$ has been applied with coefficients determined separately for each of the 7 series, except NIK TR.C, the number of observations of which is insufficient for a reliable determination (see Table 1).

4. Construction of a mean instrumental system

Reduced in this way to a common stellar reference frame and an ephemeris of comparison, all collected residuals $(O - C)_\alpha$, $(O - C)_\delta$ for a certain object and equatorial coordinate represent a combination of instrumental series. It is inhomogeneous in the general case, since each of the series is affected by a whole set of systematic errors, such as seasonal errors, errors of the adopted formulae for limb correction, used refraction model, night-day effects, and errors proper only to a certain instrument. Resulting from the individual features and quality of an instrument, atmospheric conditions of an observing site, thoroughness of the analysis of individual instrumental errors made by observers who publish their observations, systematic errors proper to a certain method are manifested differently in the results of one or another instrument. Assembled together with equal weights, observations from different instruments compose a "mean instrumental system" (MIS) which may be considered as a more reliable selection for determination of discrepancies between stellar and dynamical frames, for in this case systematic errors of individual series may be assumed as random in an external sense, according to a classical statistical concept, and should presumably suppress one another in the summation procedure. This is valid provided that we have a relatively numerous sample of series. If not, MIS may be distorted by some series which considerably deviate from the others. In this case raises the problem of a justified criterion for a weighting system that, when applied to a real configuration of series, forms MIS more-or-less close to what one could obtain by using a statistically sufficient number of series. High internal accuracy of observations could not guarantee against systematic errors and should not, hence, be accepted to be this criterion, in our opinion. More natural to use an external accuracy of a series relatively MIS, but, again, its estimation implies existence of a reliable and undistorted MIS.

The latter, nevertheless, is impossible to construct without justified external weights of instruments.

In this study, to define a weighting system for the actual combinations of instruments at our disposal, we have made use of the idea of Khrutskaya (1980), who proposed to assign a weight to a certain series taking into account its destructive influence on the MIS. To be applied to our case of residuals $(O - C)$, in the form of time series, this idea appears as follows.

Let $D_j^{(-k)}$ be the variance of deviations of the j th series against some intermediate MIS formed with the k th series being excluded. A weight for the k th instrument is then assigned to be proportional to the normalized sum of the variances of all remained series

$$P^{(k)} = \frac{\sum_j D_j^{(-k)}}{\sum_k \sum_j D_j^{(-k)}}. \quad (4)$$

Simulative experiments have shown that a MIS defined in this way appears to a considerable extent to be free from distortions caused by significantly deviated series. It is possible, therefore, to determine a more-or-less reliable external weight for a series against a MIS formed in this way.

Let $y^{(m)}$ be a series of $(O - C)$ residuals obtained with the m th instrument over the entire observational period for a certain object and equatorial coordinate, $z^{(m)}$ a smooth curve fitted to this series, and $Z^{(-m)}$ a smooth curve fitted to all observations except that m th instrument, i.e. it represents an intermediate MIS. The variances in (4) are then calculated using deviations of corresponding smooth curves, $z^{(m)}$ against $Z^{(-m)}$.

Let Z be a weighted smooth curve fitted to residuals of all instrumental series with weights assigned to each according to (4), i.e. Z represents the final MIS. If $S_i^{(m)} = D(y^{(m)} - z^{(m)})$ is the variance of residuals of the m th series against its proper smooth curve, and $S_e^{(m)} = D(y^{(m)} - Z)$ is the variance of these residuals against the final MIS, the weight in an external sense of the m th series may then be defined as $P_e^{(m)} = S_i^{(m)} / S_e^{(m)}$, while its weight in an internal sense will be $P_i^{(m)} = S_0^{(m)} / S_i^{(m)}$, S_0 being some normalizing value. It is clear that the maximum external weight 1.0 is reached when an individual instrumental system is coincident with the MIS. Such a representation of external weights determined for an object separately in right ascension and declination is convenient because, usually, conditional equations are solved for in a joint solution for both equatorial coordinates.

In this study, we have used as a smoothing procedure Vondrak's method developed for the case of non-equal weighted and non-equidistant observations (Vondrak 1969, 1977). Smoothing coefficients have been empirically adjusted for all three objects in such a way that the smoothing procedure works as a frequency filter with lower level equal to the orbital period of the object.

It should be noted that, being applied to the case of a certain selection of time series nonuniformly distributed in time, different in density, covering different intervals and sometimes having rather large lacunae (see Fig. 1a-f), this method gives as a result some MIS, representing at different intervals of time

different combinations of instrumental series actually available for analysis.

Applying the described method to the planetary observations presented in Sect. 2, we defined the weighting system of the instrumental series for each object and equatorial coordinate. Mean instrumental systems represented by the weighted smooth curves in right ascension and declination for the Sun, Mercury and Venus plotted against time are shown in Fig. 2a–f. The density of the smooth curves reflects the actual summarized density of observations collected for this study. The internal and external weights of the instrumental series in both coordinates, P_i and P_e , are presented in Tables 1–3. The relative differences between P_i and P_e of some series are instructive. The mean values of P_e computed with weights proportional to a number of observations give a general idea of the homogeneity of the actual configuration of series.

5. Analysis of the residuals

The systematic part of the weighted residuals $(O - C)_\alpha$ and $(O - C)_\delta$ represents a combined effect due to errors in orbital parameters of the comparison ephemeris, to inaccurate orientation of the reference catalogue relative to the dynamical frame of the ephemeris, and to systematic errors attributed to the method of observation and to reduction of the initial observational material. These latter are to some extent usually proper to all series and are seen in summary in MIS of the corresponding object and equatorial coordinate. Figure 2a–f and the periodograms from spectral analysis of $(O - C)_\delta$ residuals for the Sun, Mercury and Venus, and $(O - C)_\alpha$ residuals for Venus presented in Fig. 3a–d, give an idea of the actual character of these errors manifested in the modern optical observations of the Sun and inner planets. The most general features are as follows: 1) a positive linear trend in right ascension is clearly displayed for all three objects, despite the relatively short time span of the observational material under analysis. Meanwhile no apparent trend is detected in declination for all three objects; 2) for the Sun, the smooth curves have irregular character as a whole, but an annual variation in declination is clearly seen in the first half of the 30-year interval; 3) for both Mercury and Venus, the nearly annual and sidereal harmonics (0.24 year for Mercury and 0.61 year for Venus) are noticeable, being in amplitude about 0.15'' for Mercury and 0.25'' for Venus. The MIS of Venus right ascensions represents a combination of the strong harmonic with the nearly synodic period, 1.61 year, clearly seen in Fig. 2e, and a set of harmonics with periods less than 1 year having amplitudes from 0.1'' to 0.15'' seen in the periodogram (Fig. 3d). A slight negative shift is also noticeable for declination curves of Mercury and Venus (Fig. 2d and 2f), as well as apparent decreases of the amplitudes of periodic deviations in the second half of the analysed interval.

This most general analysis has shown the existence in the residuals of errors of all three types cited above. The annual harmonics may be considered as a certain mixture of the well-known annual systematic error in declination of meridian observations and errors in the obliquity of the ecliptic and in the origin

of the mean longitude, the latter two being inseparable from the former one. The synodic harmonic is due to inadequacy of the conventional formulae of phase correction. Sidereal harmonics detected for both planets give evidence that, despite the high internal accuracy in determination of the orbital elements of the inner planets in DE200 (Standish 1990), their actual errors might exceed the claimed formal ones.

6. Conditional equations

For analytical estimation of the orientation of the FK5 reference frame and corrections to elements of Earth's and planets' orbits, the conventional conditional equations suggested by S. Newcomb were applied for the Sun:

$$\begin{aligned} (O - C)_\alpha &= -\Delta E - \Delta \dot{E}(t - t_0) + \cos \varepsilon \sec^2 \delta \Delta L_0 \\ &\quad + \cos \varepsilon \sec^2 \delta \Delta \dot{L}_0(t - t_0) - \cos \alpha \tan \delta \Delta \varepsilon \\ &\quad + 2 \sin \alpha \sec \delta \Delta h - 2 \cos \alpha \sec \delta \cos \varepsilon \Delta k \\ (O - C)_\delta &= -\Delta D + \cos \alpha \sin \varepsilon \Delta L_0 \\ &\quad + \cos \alpha \sin \varepsilon \Delta \dot{L}_0(t - t_0) + \sin \alpha \Delta \varepsilon \\ &\quad + 2 \sin \delta \cos \alpha \Delta h - 2 \cos^2 \alpha \cos \delta \sin \varepsilon \Delta k \end{aligned} \quad (5)$$

where ΔE and ΔD are corrections to the equinox and equator of a reference catalogue, ΔL_0 the correction to the origin of Sun's mean longitude, $\Delta \varepsilon$ is the correction to the obliquity of the ecliptic, Δh and Δk are related to corrections to the eccentricity of the Earth's orbit e_0 and the longitude of perihelion π_0 by the equations

$$\begin{aligned} \Delta e_0 &= \Delta h \cos \pi_0 + \Delta k \sin \pi_0 \\ e_0 \Delta \pi_0 &= -\Delta h \sin \pi_0 + \Delta k \cos \pi_0 \end{aligned}$$

A secular variation of the equinox correction $\Delta \dot{E}$ and correction to the mean longitude $\Delta \dot{L}_0$ was additionally introduced into the equations, $(t - t_0)$ being the time in centuries past the mean epoch of the observations under analysis.

For Mercury and Venus observational data, we applied equations derived by Sveshnikov (1972) assuming a circular and coplanar orbit of the planet. Since, due to language difficulties, Russian editions are not always accessible to all astronomers, we quote these formulae in the Appendix.

To assess the actual reliability of the conditional equations as applied to different objects and time intervals, we have undertaken the following simulative experiment. The differences of the type $(O - C)_\alpha$ and $(O - C)_\delta$ were composed for the Sun, Mercury, Venus and Mars on the basis of the unmodified Newcombs planetary theory, as defined in (Stumpff 1981), and the intermediate ephemeris called in (Standish 1982) DE119. The latter has been formed by inverse transformation of DE200 from the epoch J2000 to the epoch B1950, applying to rectangular coordinates of the DE200 3×3 matrix given by Lieske et al. (1977)

$$\mathbf{r}_{\text{DE119}} = \mathbf{P}_{(\text{J2000} \Rightarrow \text{B1950})} \mathbf{r}_{\text{DE200}}$$

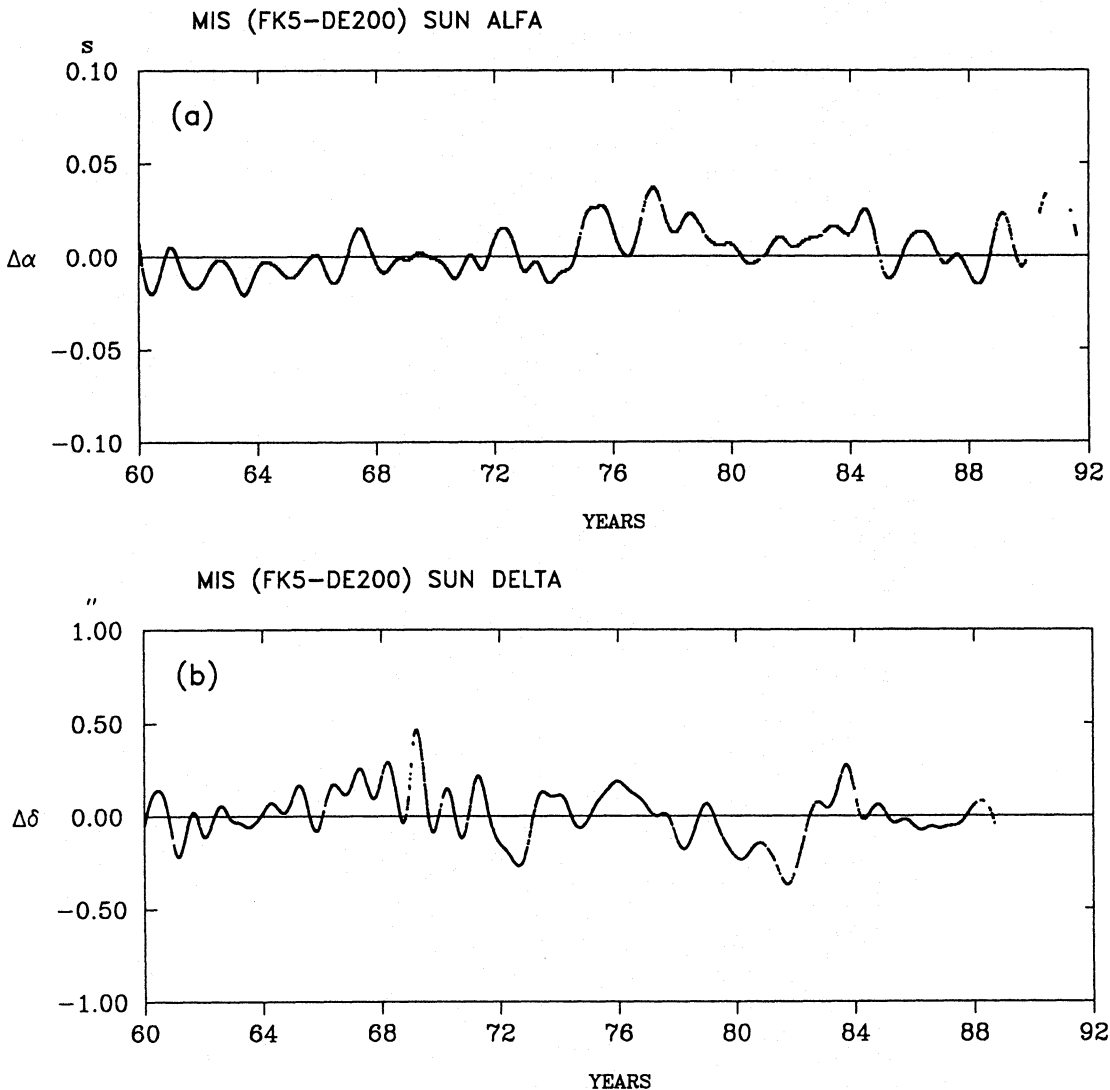
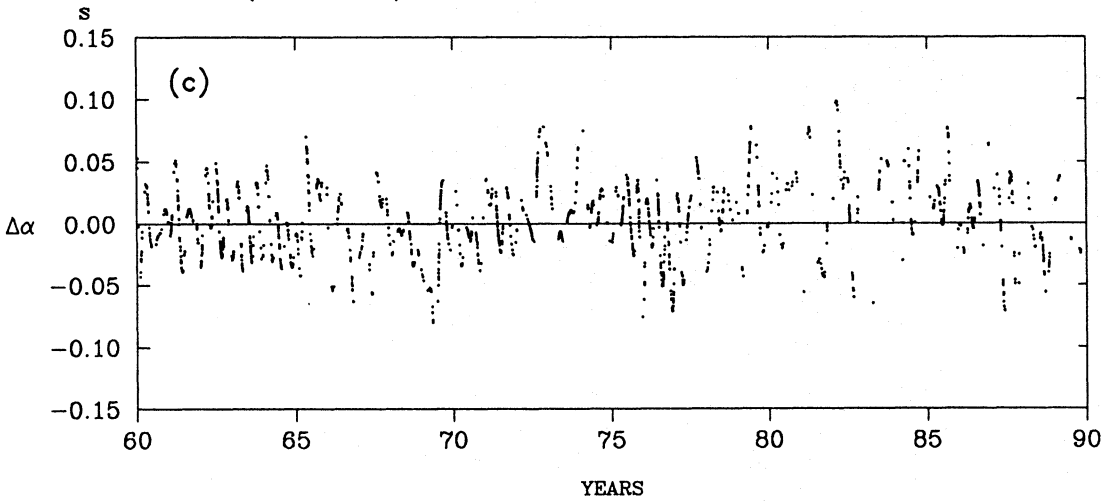


Fig. 2a-f. The mean instrumental systems of ($O - C$) residuals (MIS), represented by the weighted smooth curves in right ascension and declination respectively for the Sun (a and b), Mercury (c and d) and Venus (e and f)

Corresponding apparent places have been calculated at 5-day intervals over the time span from 1800 to 2050 in accordance with the standard procedures conventional before adoption of the IAU 1976, 1980 system. The precession and nutation matrices have been applied to rectangular equatorial coordinates r_{DE119} with Newcomb-Andoyer precession parameters and Woolard's coefficients of nutation (Explanatory Supplement the A.E. 1974), which then have been converted into equatorial polar coordinates. A set of apparent places given by the DE119 may be considered in this simulation as some ideal observational series obtained and reduced in the equatorial coordinate system. On the other hand, the apparent places of the Sun and the planets have been obtained from of-date values for ecliptic polar coordinates given by Newcomb's theory. The residuals (in the sense DE119 - Newcomb) formed in this way are presented in Fig. 4a-h, where differences in apparent right ascensions and declinations for the Sun, Mercury, Venus and Mars are plotted over the time span 250 years.

It should be noted that residuals ($\alpha_{\text{DE119}} - \alpha_{\text{NEWCOMB}}$) and ($\delta_{\text{DE119}} - \delta_{\text{NEWCOMB}}$) suit perfectly for testing of the conditional equations, for they represent the data, formed in conventional way, to which the equations of this type are usually applied, while at the same time embodying pure differences between Keplerian parameters of the ephemeris and the theory and differences between the relative orientations of their reference frames in the sense of classical differential astrometry. Positions of planets in both the ephemeris and the theory are to a considerable extent intrinsically consistent. Thus solutions obtained from different objects, in the ideal case when the equations work perfectly, should give nearly identical results on frames orientation and differences in orbital parameters of the Earth. Discordance between solutions could give evidence of influence of correlation links between coefficients of equations on final results and ultimately of the external accuracy that could be achieved when using these equations. On the other hand, these artificially-made series are free from the systematic er-

MIS (FK5-DE200) MERCURY ALFA



MIS (FK5-DE200) MERCURY DELTA

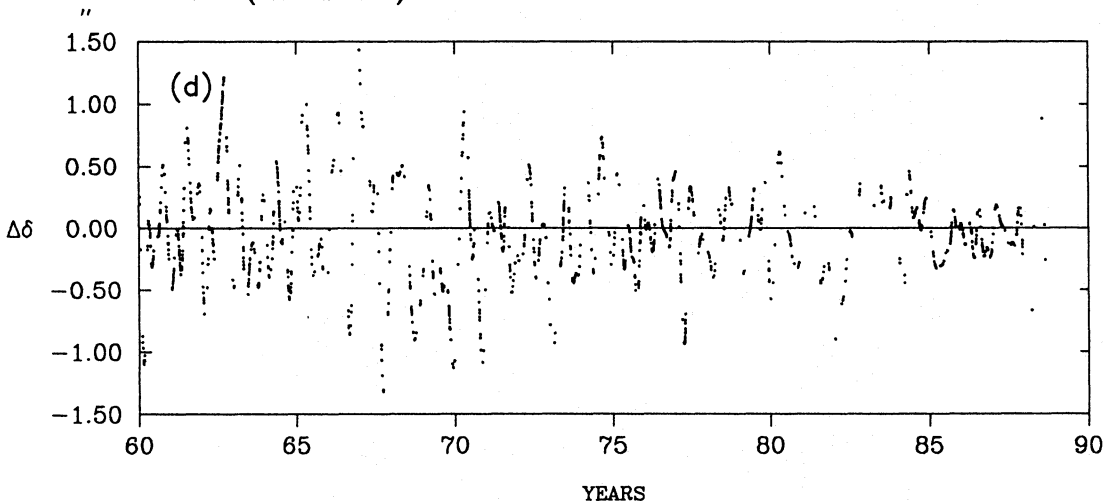


Fig. 2. (continued)

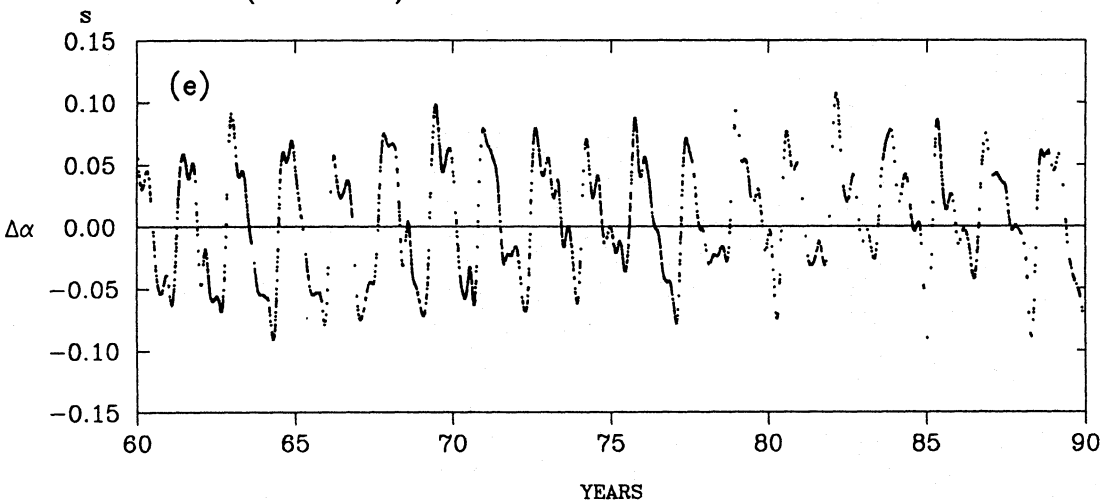
rors of observations that often significantly distort the results. Such a simulation could also give some insight into a concept of the dynamical equinox that, in practice, is inseparable from an ephemeris of comparison, a procedure of comparison and a mathematical model applied to a set of ($O - C$) residuals.

Before construction of the modern numerical ephemerides, it was a priori adopted that the dynamical equinox of an analytical planetary theory represents the best possible approximation to the inertial frame. The systems of the whole series of fundamental catalogues have been adjusted to the frame of Newcomb's theory. Now, a numerically integrated ephemeris, the DE200, has formed the basis for most of the astronomical almanacs. A method of its orientation in inertial space has been developed and realized by Standish (1982). It is, therefore, a matter of interest to assess the relative orientation of these two fundamental frames in the equatorial coordinate system and its temporal stability with regard to the methods of differential astrometry. Evidently, straightforward comparison would be untenable, since the ephemeris from which DE200 originates

has been precessed from the epoch B1950 to the epoch J2000 with the IAU 1976 precession and obliquity, and, hence, its equinox from this step could not be considered as completely independent from the system of constants. But, if one wishes to interpret differences between of-date positions given by an ephemeris and Newcomb's theory, these should be formed with Newcomb's value of precession, which is implicitly embodied in of-date values for ecliptic polar coordinates generated by this theory. As to DE119, the equinox realized in this intermediate ephemeris is intrinsically based only upon the formalism of the procedure proposed by Standish.

It is clear that, in this case, the "equinox correction" ΔE , its secular variation and the "equator correction" ΔD from LS solutions, will have to a considerable extent only a formal sense connected with a mathematical model under analysis which defines correlation links between coefficients of unknowns and, hence, the real accuracy of their determination. Their numerical values would give such differences in equinox and equator estimates which may be obtained from real observations when one

MIS (FK5-DE200) VENUS ALFA



MIS (FK5-DE200) VENUS DELTA

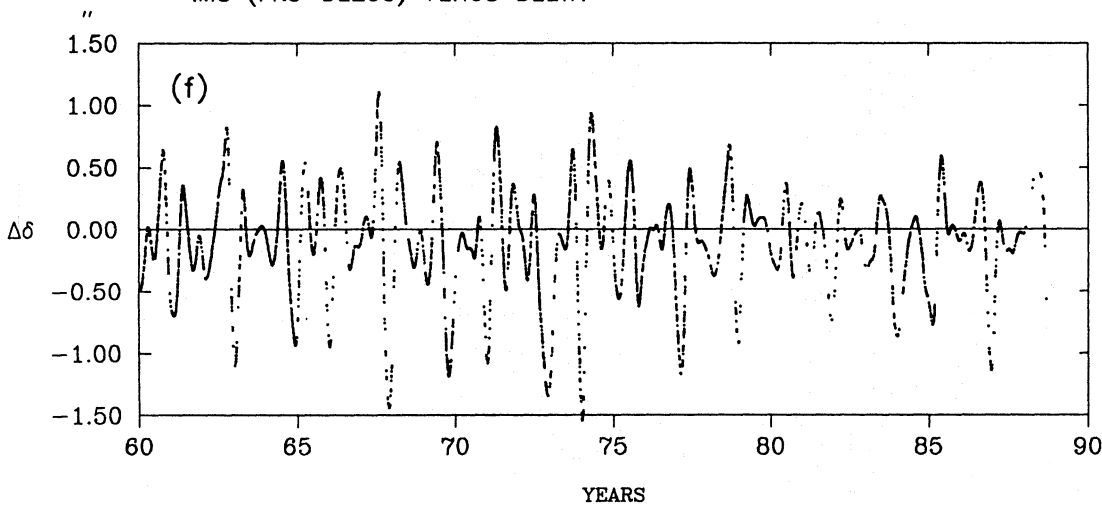


Fig. 2. (continued)

changes an ephemeris of comparison, but keeps the astronomical constants used in computation of apparent places unchanged.

A matter of a special concern in this paper is the secular equinox drift. To estimate it properly and separate it from analogous variations of other parameters, the secular terms $\Delta \dot{E}$, $\Delta \dot{L}_0$, and $\Delta \dot{L}$, representing non-precessional equinox motion and corrections to the of-date mean motions of the Earth and planet, have been additionally included into Eq. (A1).

The results of LS solutions with conditional equations presented above for the three time intervals, 1800–2050, 1910–1990, 1960–1990, are listed in Table 4. The second interval is chosen as representing the span where most of the optical planetary observations were collected after invention of the impersonal micrometer and could potentially be used for analysis. The third one is the interval under analysis in the present study. Despite the fact that we are not concerned here with Mars, it is also included into the comparison, because its observations sometimes are used for the equinox and equator determinations,

and it is a matter of interest to assess how conditional equations work with this object.

Analysis of Table 4 leads to the following conclusions. Solutions obtained for Mars deviate sharply from these for the three other objects. It seems that this planet is the least suitable object for determination of the orientation of frames. Henceforth, Mars is excluded from the discussion. The most consistent solution is obtained for ΔL_0 from all three intervals and objects. Equinox corrections from the residuals of all three objects are consistent within $0.01''$ and their secular rates within $0.03'' \text{ cy}^{-1}$ for 250-year and 90-year intervals. For 30-year interval discrepancies attain $0.02''$ for ΔE and $0.1'' \text{ cy}^{-1}$ for $\Delta \dot{E}$. It is seen also that the equinoxes of DE119 and Newcomb's theory do not coincide, a slight secular variation of the equinox difference (about $0.02'' \text{ cy}^{-1}$ from the 250-year interval) having been detected as a result of the comparative simulation. When comparing the values of $\Delta \dot{L}_0$ derived from different intervals, a quick degradation of their estimates at the shorter intervals is noticeable, so that one can expect deviations of about $0.2'' \text{ cy}^{-1}$ from real

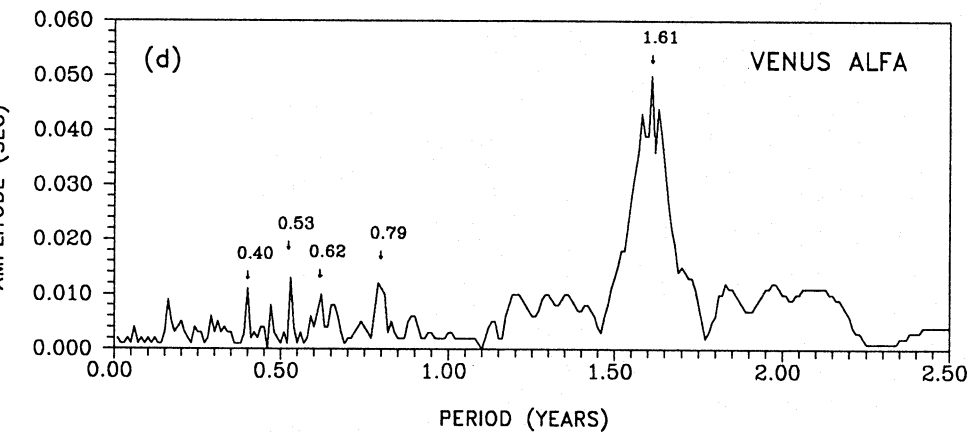
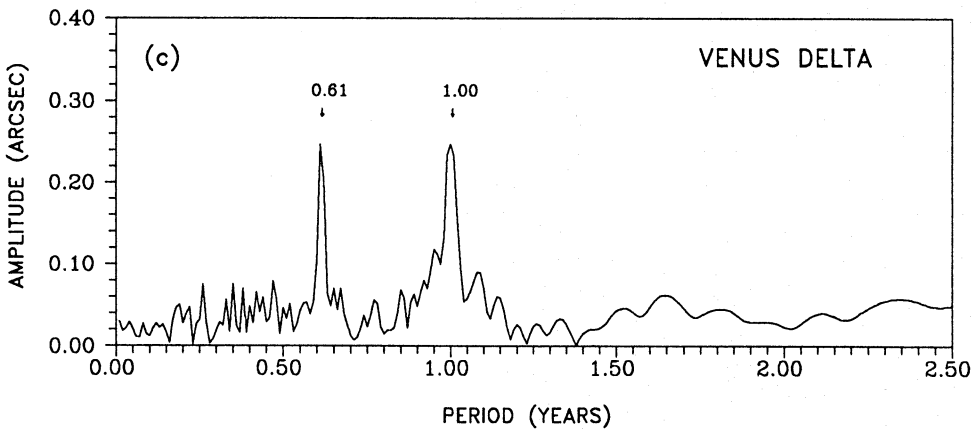
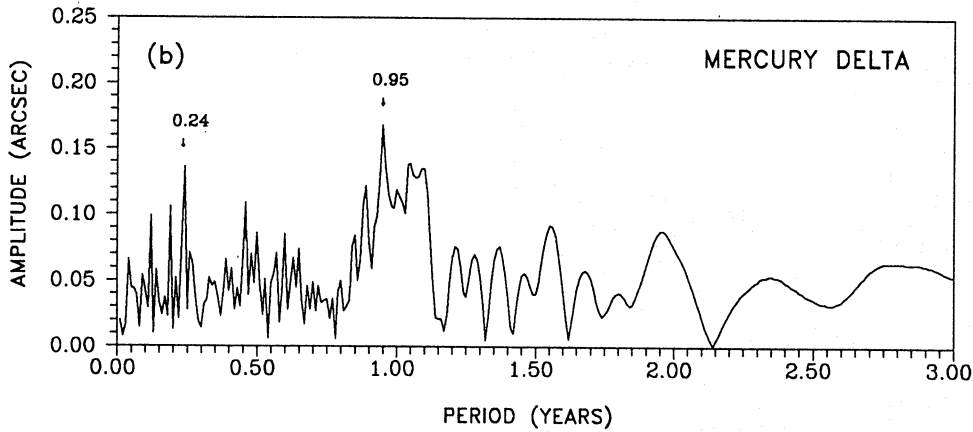
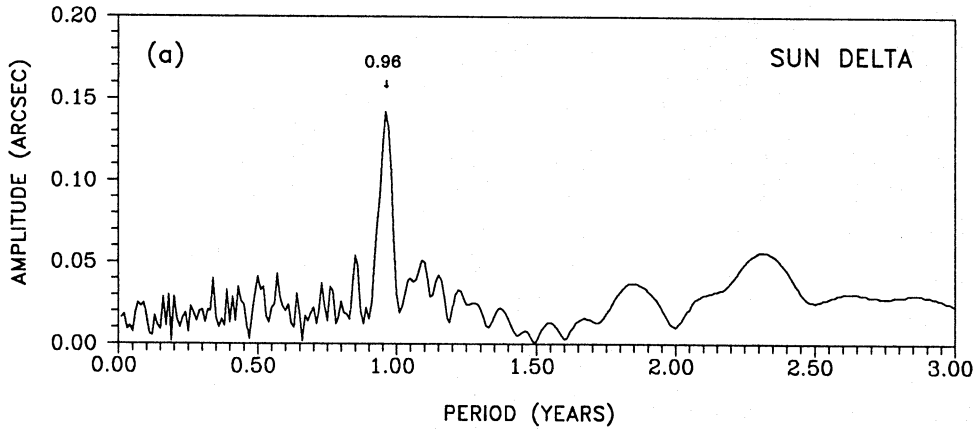


Fig. 3a-d. Periodograms of $(O - C)_\delta$ residuals for The Sun (a), Mercury (b), Venus (c), and of $(O - C)_\alpha$ residuals for Venus (d)

values when solutions are taken from the 30-year interval. The mean result on $\Delta\dot{L}_0$ from all three objects at 250-year interval, $-0.041'' \text{ cy}^{-1}$, may be compared with that by Stumpff & Lieske (1984) [taking into account the difference between inertial mean motions of DE102 and DE118 given by Newhall et al. (1983)], confirming their conclusion that the of-date real mean motion of the Earth-Moon barycenter given by the modern tabular ephemerides is comparable with that given by Newcomb's planetary theory only by using Newcomb's precession.

Taking values of ΔE and $\Delta\dot{E}$ determined as a mean for all three objects from the 250-year interval, we obtain the difference $0.038''$ to be the mutual orientation of the frames in the equatorial system on the epoch B1950. From this result follows the conclusion: dynamical equinoxes of DE119 and Newcomb's theory, determined with the typical conditional equations of differential astrometry, are nearly identical on the epoch of modern observations and the secular variation of their mutual orientation is negligible compared with values usually obtained from optical observations. In the procedure by Standish (1982), DE119 has been transformed to the epoch J2000 by using IAU 1976 expressions for the lunisolar precession. We abstain here from any suggestions on the equinox location after this transformation, which needs additional investigation.

Comparing solutions for ΔE , $\Delta\dot{E}$, ΔL_0 , $\Delta\dot{L}_0$, ΔD , $\Delta\varepsilon$, Δe_0 , $e\Delta\pi_0$, resulting from residuals of the Sun and inner planets taken at different intervals, one can assess something like an external accuracy of the conditional equations presented in this section. It is natural that the most consistent results are derived from the 250-year interval. Differences in estimates of all parameters are not greater than $0.02''$ in absolute value. Consistency between solutions for constant terms obtained from the three objects does not significantly degrade at the shorter intervals. The inconsistency of secular variations, however, might reach $0.1'' \text{ cy}^{-1}$ at the 30-year interval. When comparing solutions from different intervals, the results might differ by about $0.05''$ for the constant terms, $0.1'' \text{ cy}^{-1}$ for the secular variation $\Delta\dot{E}$ and $0.2'' \text{ cy}^{-1}$ for $\Delta\dot{L}_0$. It should be noted that this estimate of external accuracy given by the conditional equations was made on the basis of "ideal" simulated data a priori free from systematical errors of real observations and, thus, may be considered as to be the most optimistic. This should be born in mind when analysing the results obtained from the real observational data.

As for Keplerian orbital parameters of Mercury and Venus, the inconsistency in their estimates from different intervals are much greater. Some of them, such as ΔL and $\Delta\Omega \sin I$, differ by $0.5''\text{--}1''$ when comparing 250-year and 30-year intervals. Interpretation of this is beyond the scope of our paper, in which we are concentrating on the frame orientations. We would, however, emphasize that, this ultimately does not affect the results on frames orientation and on parameters of the Earth's orbit.

Applying the conditional equations described in this section to the observational material under analysis, we have made the following modifications. For the Sun, considering that the coefficient of ΔL_0 in right ascension Eqs. (5) is quasi-constant, so that the secular variations of ΔE and ΔL_0 are inseparable for

this equatorial coordinate, we determine here $\Delta\dot{L}_0$ exclusively from the declination part of Eqs. (5). For this, we have applied an iterative method, solving the system (5), modified as follows,

$$\begin{aligned} (O - C)_\alpha - (\cos \varepsilon \sec^2 \delta \Delta\dot{L}_0 (t - t_0) + \cos \varepsilon \sec^2 \delta \Delta L_0 \\ - \cos \alpha \tan \delta \Delta \varepsilon) \\ = -\Delta E - \Delta\dot{E} (t - t_0) + 2 \sin \alpha \sec \delta \Delta h \\ - 2 \cos \alpha \sec \delta \cos \varepsilon \Delta k \end{aligned} \quad (5a)$$

$$\begin{aligned} (O - C)_\delta - (2 \sin \delta \cos \alpha \Delta h - 2 \cos^2 \alpha \cos \delta \sin \varepsilon \Delta k) \\ = -\Delta D + \sin \alpha \Delta \varepsilon + \cos \alpha \sin \varepsilon \Delta L_0 \\ + \cos \alpha \sin \varepsilon \Delta\dot{L}_0 (t - t_0) \end{aligned} \quad (5b)$$

separately in right ascension and declination. First, Eq. (5a) are solved with ΔL_0 , $\Delta\dot{L}_0$ and $\Delta\varepsilon$ being zero. Solutions for Δh and Δk are then introduced into the left-hand side of Eq. (5b) in the second step, from which solutions for ΔL_0 , $\Delta\dot{L}_0$ and $\Delta\varepsilon$ are obtained and applied to the left hand side of Eq. (5a). Iterations continue until the convergence of results is achieved.

For observations of Mercury and Venus, we have used Eq. (A1). These latter equations have been complemented with two terms, written separately for the upper and lower conjunction sectors of an orbit, which evaluate the deficiency of the conventional limb corrections, applied in the primary reductions of planet observations:

$$\left| \frac{\Delta f_{\alpha,1}}{\Delta f_{\delta,1}} \right| = \left| \frac{\sin \theta}{\cos \theta} \right| \Phi_1 \cos i \quad (i > 90^\circ) \quad \text{and}$$

$$\left| \frac{\Delta f_{\alpha,2}}{\Delta f_{\delta,2}} \right| = \left| \frac{\sin \theta}{\cos \theta} \right| \Phi_2 \sin i \quad (i < 90^\circ) .$$

The coefficients Φ_1 and Φ_2 are solved for in the solutions of Eq. (A1). Here i is the phase angle of the planet, and θ is the position angle of the midpoint of the illuminated edge.

Assuming corrections to the mean motions of the inner planets in DE200 to be negligible, for Venus data in the solution of (A1) we added only the secular variations of ΔE and ΔL_0 . The approximate mean epoch for Sun and Venus solutions was taken to be 1973.0. The estimates of the respective secular parameters for Mercury have yielded rather unrealistic results: $\Delta\dot{L}_0 = (-1.317'' \pm 0.640'') \text{ cy}^{-1}$ and $\Delta\dot{E} = (-1.818'' \pm 0.595'') \text{ cy}^{-1}$, the correlation between their coefficients being 0.89. We ascribe this result to the particular character of the observational material over the relatively short time span taken for analysis and are thus limiting here solutions to only constant terms for this planet.

7. Results of the least squares solutions and discussion

The results of the weighted solutions (with weights in an external sense P_e assigned to the instrumental series according to the method described in Sect. 4 and presented in Tables 1–3) obtained from the Sun, Mercury and Venus observations are given in Table 5, where all notations of the corrections to the

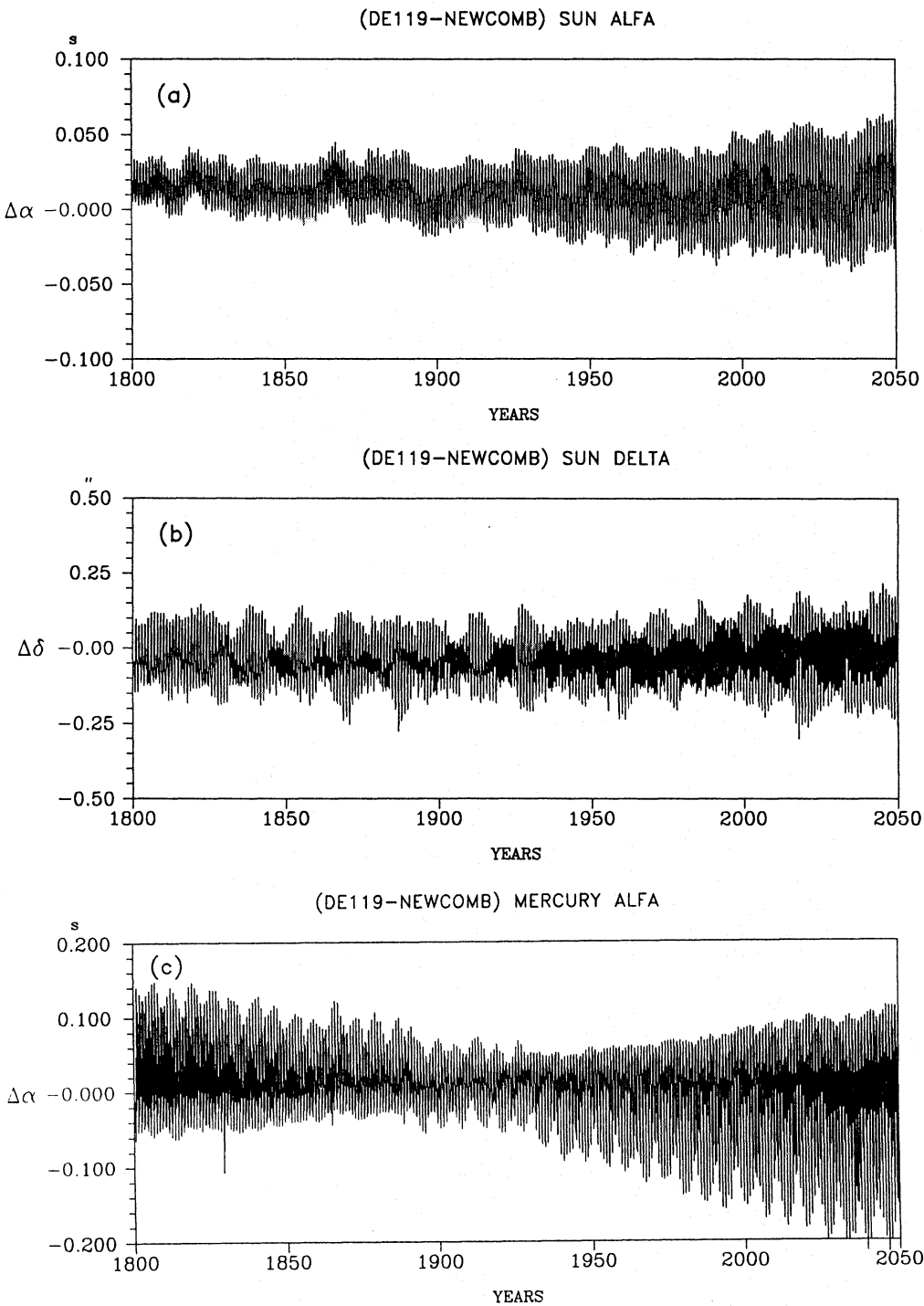


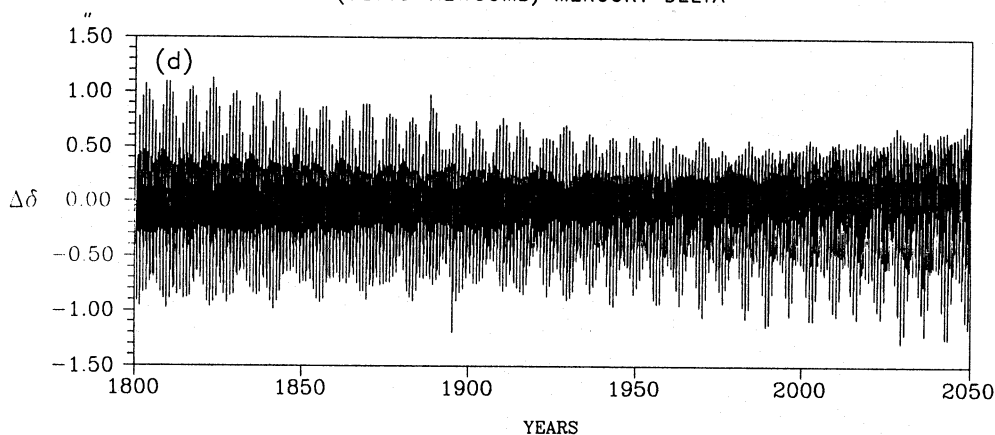
Fig. 4a-h. Residuals in right ascension and declination on a 250-year interval between apparent places computed with the standards IAU 1964 on the basis of the ephemeris DE119 and Newcomb's theory for the Sun (a and b), Mercury (c and d), Venus (e and f), Mars (g and h)

frames orientation and to the orbital parameters are defined in the previous section.

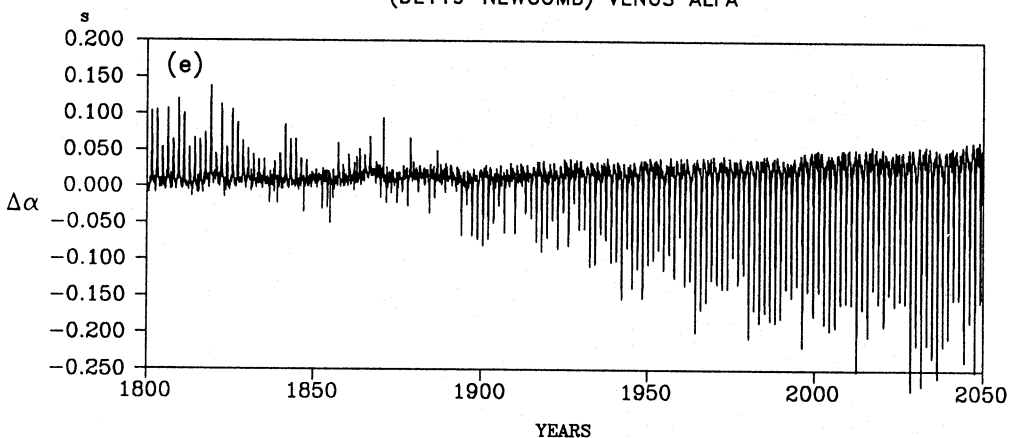
It should be noted that, despite a much larger number of observations, the formal precision of the corrections is not far superior to those from separate individual series. We see, however, the evident advantages of our global approach in the reliability of the results. Their consistency is comparable with the

results obtained in the simulations of the previous section. One can notice that the corrections to the Earth's orbital parameters and to the orientation parameters of the FK5 derived from both Mercury and Venus solutions are very close at a level of consistency rarely achieved from observations made with one instrument. This, in our opinion, confirms our initial assumption on the mutual suppression of individual systematic errors

(DE119-NEWCOMB) MERCURY DELTA



(DE119-NEWCOMB) VENUS ALFA



(DE119-NEWCOMB) VENUS DELTA

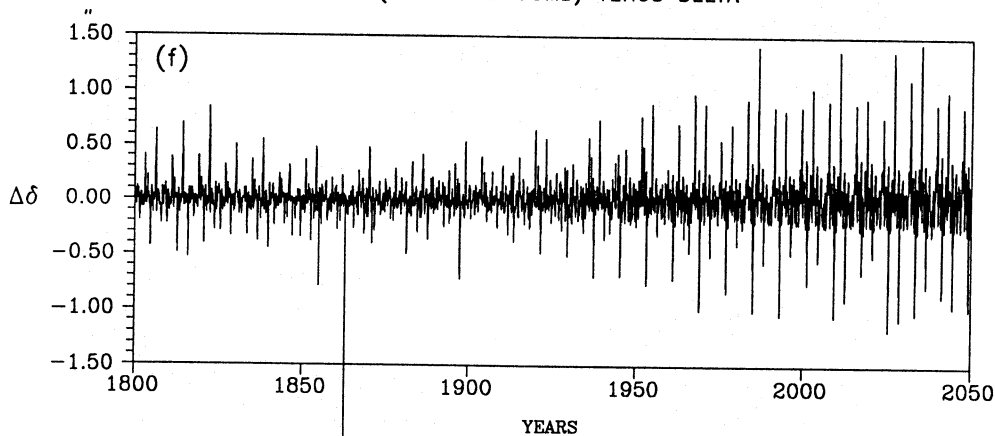


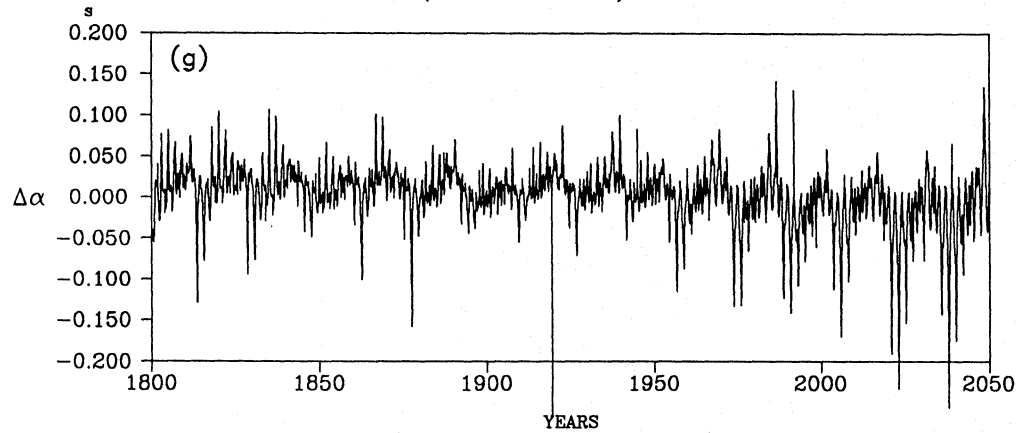
Fig. 4. (continued)

of observations in a joint weighted solution. Only the $\Delta\varepsilon$ for the planets are significantly different with regard to their formal errors.

The Sun's results are rather different for several parameters ($\Delta\varepsilon$, ΔD , $e_0\Delta\pi_0$) but ΔL , ΔE , $\Delta\dot{L}_0$ and $\Delta\dot{E}$ corrections are consistent with the planetary ones. As for Sun's ΔD , it is strongly affected by the large positive shift of $(O-C)_\delta$ residuals

in the sixties (see Fig. 2b), which is absent in the corresponding plots of the planets' $(O-C)_\delta$. The Sun's solution based on the period 1975–1990 gives a positive ΔD correction $0.04''$ which is quite near to the planets' mean value $0.06''$. This systematic difference of ΔD estimates (about $0.1''$) between results obtained from the Sun and inner planets during the entire 20-th century is effectively illustrated in (Sveshnikov 1985), Fig. 1b.

(DE119-NEWCOMB) MARS ALFA



(DE119-NEWCOMB) MARS DELTA

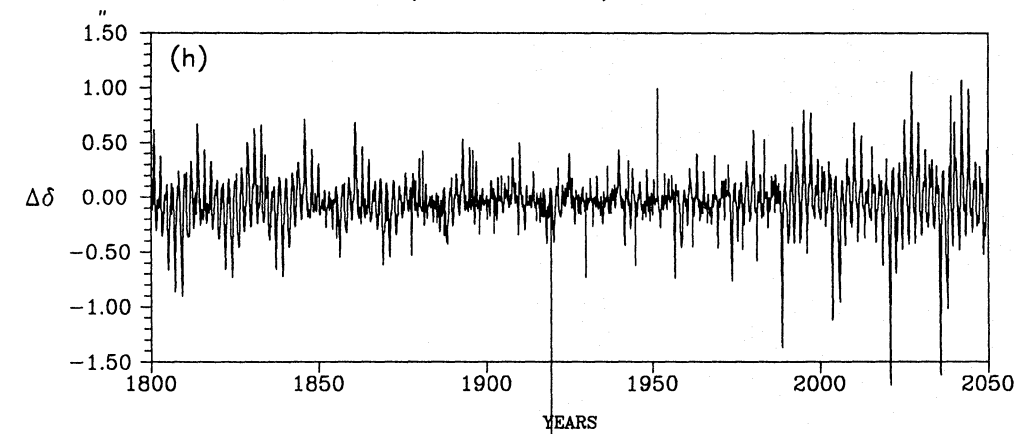


Fig. 4. (continued)

Nearly the same result has been recently obtained by Yao & Smith (1993). We are inclined to adopt the supposition, expressed in the latter paper, on the strong and constant influence on the Sun's results of heating of the tube and anomalous refraction within it. It seems that this kind of systematic error, nearly equal in magnitude and in sign, affected the observed declinations of the Sun in almost all instrumental series forming MIS in the sixties and the beginning of the seventies, see Fig. 2b, while they are apparently reduced in the new series, such as GOL V.C, BELGR TR.C, KISL V.C in the second half of the analysed interval. A positive correction to the equator of the FK5 on the same order as in this study has recently been found by different authors, the most confidential of which, as based on large observational material, is Niimi's (1990) correction $0.05''$.

His result for $\Delta\varepsilon$, about $0.07''$ at the mean epoch 1973, is also quite close to ours obtained from Mercury and Venus, but this value is dramatically different from the obliquity of the ecliptic found dynamically in DE200: $-0.036''$ (Standish 1982), and in VSOP82: $-0.039''$ (Bretagnon 1982). On the other hand, our solution based on observations of the Sun is more consistent with these values, as well as with the Holdenried et al. (1993) result for the Sun, $-0.023''$. As usual, corrections to the obli-

quity of ecliptic obtained by different authors basing on observations of different objects are discrepant, ranging about $0.2''$ in magnitude. In the case of the Sun, the term containing $\Delta\varepsilon$ in declination is inseparable from the annual systematic error of meridian observations. Application of the empirical correction of the type $(A + B \operatorname{tg} z)$, as was made by Niimi (1990) and in the present study, represents a certain compromise, since coefficients of $\Delta\varepsilon$ and $\operatorname{tg} z$ are usually strongly correlated, and the real value of $\Delta\varepsilon$ might partially be absorbed by this empirical correction when subtracting from residuals. It should be noted that, for inner planets, a coefficient of $\Delta\varepsilon$ usually correlates with that for ΔI . In the case of Venus data used in this study, this correlation attains 0.7, which may explain the deviation of $0.05''$ relative to the result from Mercury. For all these reasons, we consider determinations of $\Delta\varepsilon$ by the methods of differential astrometry to be much less reliable comparing with the dynamical method cited above, which gives accuracy of the obliquity determination better than 2 mas.

As seen from Table 5, the equinox corrections resulting from all solutions are nearly identical. Its anomalous negative secular rate of change determined from Sun and Venus solutions, expected to be close to zero in the FK5 system, is the most surpris-

Table 4. LS solutions with Eq. (5) for the Sun and with Eq. (A1) for Mercury, Venus and Mars resulting from the series of residuals between apparent places computed from DE119 and Newcomb's theory taken for the time spans 1800–2050 (mean epoch 1925), 1910–1990 (mean epoch 1950) and 1960–1990 (mean epoch 1975). The notations of the corrections are defined in Sect. 6 and in the Appendix. The secular variations are expressed in arcsec cy^{-1}

	SUN		MERCURY		VENUS		MARS	
1800-2050								
ΔE	$-0.042'' \pm 0.004''$		$-0.049'' \pm 0.005''$		$-0.037'' \pm 0.005''$		$-0.180'' \pm 0.024''$	
$\Delta \dot{E}$	0.031	0.005	-0.005	0.007	0.015	0.006	-0.543	0.015
ΔL_o	0.116	0.004	0.101	0.005	0.106	0.004	-0.032	0.024
$\Delta \dot{L}_o$	-0.017	0.005	-0.068	0.007	-0.038	0.006	-0.591	0.016
ΔD	-0.003	0.001	0.007	0.002	-0.026	0.001	0.010	0.003
$\Delta \varepsilon$	-0.029	0.002	-0.045	0.002	-0.018	0.002	-0.019	0.009
Δe_o	0.123	0.001	0.103	0.003	0.122	0.003	0.277	0.011
$e_o \Delta \pi_o$	0.116	0.001	0.124	0.003	0.106	0.003	0.218	0.011
ΔL			0.239	0.008	0.556	0.005	-0.137	0.022
$\Delta \dot{L}$			2.046	0.010	0.719	0.007	-0.759	0.014
ΔI			1.844	0.011	0.123	0.003	0.015	0.004
Δe			-0.450	0.009	-0.206	0.004	0.089	0.006
$e \Delta \pi$			0.237	0.009	-0.046	0.004	0.042	0.006
$\Delta \Omega \sin I$			1.861	0.028	-0.017	0.006	-0.133	0.019
1910-1990								
ΔE	-0.036	0.003	-0.030	0.005	-0.024	0.005	-0.118	0.024
$\Delta \dot{E}$	0.031	0.013	0.002	0.020	0.004	0.019	-0.511	0.045
ΔL_o	0.088	0.003	0.084	0.005	0.086	0.004	-0.005	0.024
$\Delta \dot{L}_o$	-0.104	0.012	-0.148	0.019	-0.132	0.018	-0.549	0.049
ΔD	-0.003	0.001	0.004	0.002	-0.029	0.001	0.014	0.003
$\Delta \varepsilon$	-0.026	0.001	-0.036	0.002	-0.003	0.002	-0.017	0.009
Δe_o	0.135	0.001	0.104	0.003	0.131	0.003	0.262	0.011
$e_o \Delta \pi_o$	0.119	0.001	0.112	0.003	0.120	0.003	0.253	0.011
ΔL			0.715	0.007	0.755	0.005	-0.041	0.022

ing result of this study. It should be noted that its absolute value is close to Fricke's correction for non-precessional equinoctial motion of the FK4 system, but with a negative sign. The positive drift of right ascension residuals ($0.722'' \pm 0.268''$) cy^{-1} is also detected for Mercury when the secular term $\Delta \dot{\alpha}(t - t_0)$ is introduced into Eq. (A1). Let us consider other results.

Since the introduction of the FK5 as a fundamental catalogue and DE200 as a fundamental ephemeris, new corrections to the equinox of the FK5 have been obtained by several authors. Some of them are derived from relatively short instrumental series (Journet 1986; Sadzakov et al. 1991; Branham & Sanguin 1992) which are partly included in our database. Other ones are

Table 4. (continued)

	SUN		MERCURY		VENUS		MARS	
$\Delta\dot{L}$			2.261	0.027	0.949	0.008	-0.937	0.043
ΔI			1.900	0.011	0.156	0.003	0.035	0.004
Δe			-0.422	0.009	-0.216	0.004	0.050	0.006
$e\Delta\pi$			0.276	0.009	-0.094	0.004	0.028	0.006
$\Delta\Omega\sin I$			1.391	0.027	-0.066	0.006	0.037	0.019
<hr/>								
1960-1990								
ΔE	-0.021	0.003	-0.007	0.008	-0.004	0.007	-0.068	0.036
$\Delta\dot{E}$	-0.020	0.036	-0.078	0.083	-0.113	0.081	-0.333	0.175
ΔL_O	0.068	0.003	0.069	0.007	0.068	0.007	-0.014	0.036
$\Delta\dot{L}_O$	-0.169	0.035	-0.246	0.080	-0.230	0.078	-0.040	0.191
ΔD	-0.003	0.001	0.000	0.003	-0.029	0.002	0.015	0.005
$\Delta\varepsilon$	-0.018	0.001	-0.024	0.003	0.014	0.003	-0.014	0.014
Δe_O	0.135	0.001	0.094	0.004	0.130	0.004	0.245	0.017
$e_O\Delta\pi_O$	0.160	0.001	0.133	0.004	0.159	0.004	0.346	0.017
ΔL			1.301	0.012	0.988	0.008	-0.091	0.033
$\Delta\dot{L}$			2.534	0.117	0.487	0.085	-1.091	0.168
ΔI			1.971	0.017	0.185	0.005	0.050	0.006
Δe			-0.436	0.014	-0.232	0.007	-0.012	0.009
$e\Delta\pi$			0.302	0.015	-0.108	0.007	0.033	0.009
$\Delta\Omega\sin I$			0.950	0.043	-0.108	0.010	0.208	0.028

obtained from long series reduced to the “improved FK4 system” by adding to right ascensions Fricke’s equinox correction $0.5035 + 0.5085(T - 19.50)$ (Niimi 1990; Yao & Smith 1993; Holdenried et al. 1993), or by applying the transformation of Aoki et al. (1983) (Jordi & Rosselló 1987). In all these studies, a secular term for the drift of the equinox correction was included into the conditional equations. And this is justified, since the FK5 system is not now recognized as a quasi-inertial one, but as some kind of a kinematic system, based upon adopted values of several parameters, once introduced into proper motions of the fundamental catalogue, which are not considered to be ideal and which require corrections (Kovalevsky 1991). These are the constants of precession, nutation and the non-precessional motion of the equinox. The latter parameter, introduced into the FK4 proper motion system when constructing the FK5, the nature of which still unexplained, is the most controversial. Its

numerical values as determined by different authors range significantly: from zero (Duma et al. 1980) to somewhere close to $1.8'' \text{cy}^{-1}$ from solutions of the integrated DE111, see Table 4 in (Standish 1990).

It is known that this was done to conform the new fundamental system with results of a) the analysis of FK4 proper motions of the distant stars separately in right ascension and declination; b) the analysis of lunar occultations on the 150-year interval; c) the analysis of a certain selection of individual equinox corrections made in the 20-th century. So, a fictitious equinox motion having been introduced, it was supposed that the FK5 system would be free from the non-precessional rotation relative to a dynamical equinox, and, when processed in this system or transformed to it, all the above cited observational evidence would presumably give results for this rotation close to zero. Let us discuss them.

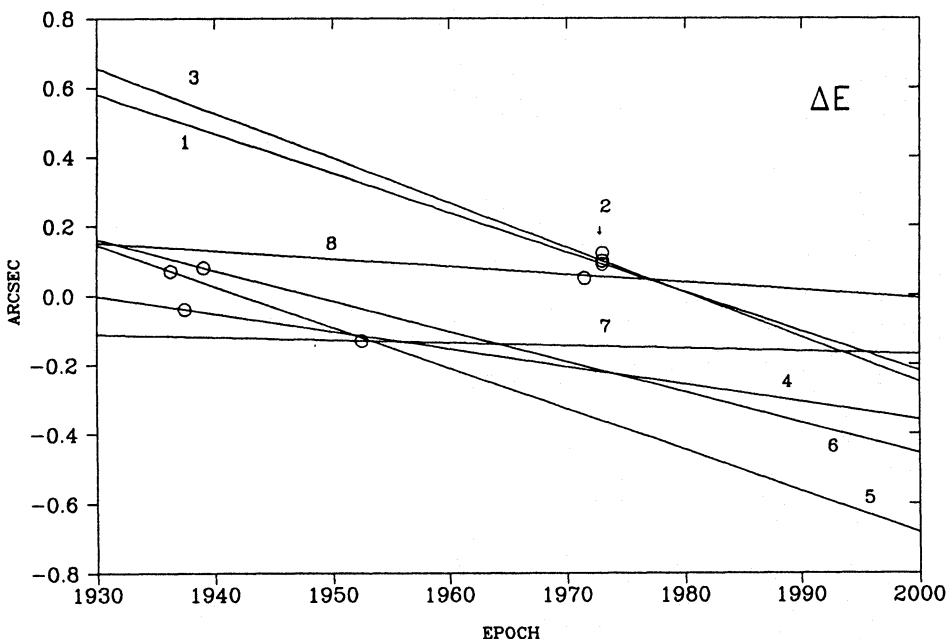


Fig. 5. Equinox corrections ΔE (circles), plotted for the mean epoch of the used observational material, and their secular rates of change $\Delta \dot{E}$ (straight lines) obtained for the Sun (1), Mercury (2) and Venus (3) in the present study; for the Sun (4), Mercury (5) and Venus (6) obtained by Yao & Smith (1993); for the innermost planets (7) obtained by Niimi (1990); and the result by Jordi & Rosselló (1987) from lunar occultations (8)

In the study of the FK4 distant stars (Fricke 1967), the determination of $\Delta \dot{E}$ linearly depended on the derived correction to the general precession in right ascension ($\Delta p \cos \varepsilon - \Delta \lambda$). Factually, this parameter was formally introduced into the solution to compensate for the sharp discordance of the derived correction to the precession constant that otherwise was obtained separately from declination and right ascension, having nearly identical values of about $1'' \text{ cy}^{-1}$ with different signs. The most recent revision of the IAU 1976 precession constant obtained independently with lunar laser ranging and an interferometric technique by Herring et al. (1991) ($-0.32'' \pm 0.13''$) cy^{-1} , Williams et al. (1991) ($-0.27'' \pm 0.04''$), Steppe et al. (1991) ($-0.276'' \pm 0.016''$) gives rather consistent results, as summarised in (Fukushima 1991) and estimated to be ($-0.25'' \pm 0.06''$) cy^{-1} . This means that the FK5 proper motion system must have an additional rotation of about $-0.2'' \text{ cy}^{-1}$ caused by overcorrection of the FK4 proper motions. A negative $\Delta \dot{E}$ correction of the same order has been obtained from analysis of lunar occultations reduced to the FK5 system and compared with DE200 (Jordi & Rosselló 1987) [this result was earlier predicted by Morrison (1982)], and from analysis of ACRS proper motions (Miyamoto & Sôma 1993). Meantime, an analysis of FK5 proper motions of distant stars similar to Fricke's made by Schwan (1988), has not revealed any significant deviations in the precession of the IAU 1976 system. The inconsistency of these results is still unexplained, see discussion in (Schwan 1990).

Results for the equinoctial motion of the "improved" FK4 system basing on optical planetary observations have been obtained by Niimi (1990) and by Yao & Smith (1993). These are plotted in Fig. 5, together with the results of the present paper and with those of Jordi & Rosselló (1987). Estimations of $\Delta \dot{E}$ in these two studies are based on observations covering nearly the same time span, but their results are dramatically

Table 5. Corrections to the equinox and equator of the FK5, to the elements of Earth's and planets' orbits and phase corrections obtained as weighted LS solutions from optical observations of the Sun, Mercury and Venus over the time span 1960–1990. The secular variations are expressed in arcsec cy^{-1}

	SUN	MERCURY	VENUS
ΔE	$0.090'' \pm 0.010''$	$0.121'' \pm 0.067''$	$0.099'' \pm 0.038''$
$\Delta \dot{E}$	-1.140 0.111		-1.292 0.251
ΔL_o	0.086 0.030	0.140 0.063	0.138 0.036
$\Delta L'_o$	0.055 0.030		0.038 0.188
ΔD	-0.002 0.009	0.054 0.020	0.071 0.012
$\Delta \varepsilon$	-0.015 0.013	0.054 0.029	0.099 0.016
Δe_o	-0.007 0.007	0.005 0.040	0.017 0.024
$e_o \Delta \pi_o$	0.005 0.007	-0.037 0.040	-0.041 0.024
ΔL		-0.661 0.162	0.071 0.042
ΔI		0.084 0.152	-0.094 0.029
Δe		-0.048 0.143	0.046 0.039
$e \Delta \pi$		0.192 0.142	0.048 0.039
$\Delta \Omega \sin i$		-1.186 0.429	0.006 0.053
Φ_1		-1.310 0.177	-1.720 0.044
Φ_2		0.470 0.031	0.956 0.021

different. Niimi has not found any significant motion of the equinox, meanwhile Yao & Smith found it to be somewhere close to $-0.85'' \text{ cy}^{-1}$, as a mean, using two long series of observations in USNO and Cape centred at epoch 1937.7. This is all the more surprising taking into consideration the fact that individual equinox corrections obtained in the FK4 system from observations in these two observatories account for around 50% of the totality of corrections used by Fricke (1982) to derive the equinox motion of the FK4.

Our result for the equinox motion, obtained on the basis of the most modern observational material including a large number of instrumental series, is in good consistency with that by Yao & Smith. We can now suggest the following factors to be the cause of this unexpected anomalous value of $\Delta\dot{E}$. This might be due to 1) error in the inertial mean motion of the Earth; 2) systematic errors of observations varying with time; 3) a constant error in proper motions of the FK5.

The first assertion seems to be the least realistic. It was proved by Standish & Williams (1990) that the error in the mean motion of the Earth in DE200, which is adjusted to the range data, cannot be as much as $1'' \text{ cy}^{-1}$. Our result on the secular variation of the difference between the mean longitude of the Earth in DE119 and Newcomb's theory from simulation in the previous section may serve as indirect evidence of this.

The second assertion might be a very possible explanation of the positive drift of the right ascension residuals. The time span of the observations actually used being relatively short, the drift could be caused by a disbalanced configuration of the available instrumental series at different intervals, as it was noted in Sect. 2. It is hard to imagine, however, any systematic error, affecting all observational series forming MIS in the eighties, which could produce a shift nearly equal in magnitude and having equal sign in the apparent right ascensions of all three objects.

The third assertion leads to a rather extravagant hypothesis that the modern optical observations of the Sun and inner planets do not confirm existence of the non-precessional equinoctial motion detected in the FK4 and introduced into the fundamental proper motions when constructing the FK5.

It is too early to make any definitive conclusions because the results obtained on the equinox motion of the FK5 by other authors are still contradictory, as seen from Fig. 5. They are based on different selections of observational material, different methods to reduce observations to the FK5 system, and different conditional equations. We hope the question could be clarified if the method of analysis used in this study were applied to a larger mass of observational data obtained with different instruments in the 20-th century.

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Appendix

Here are presented the conditional equations extracted from the paper (Sveshnikov 1972), which are used in this study

$$(O - C)_\alpha \cos \delta = a_1 \Delta \lambda^0 + a_2 \Delta p + a_3 \Delta q + a_4 \Delta u \\ + a_5 \Delta \nu + a_6 \Delta \Lambda_0 + a_7 \Delta P_0 \\ + a_8 \Delta Q_0 + a_9 \Delta U_0 + a_{10} \Delta V_0$$

$$(O - C)_\delta = b_1 \Delta \lambda^0 + b_2 \Delta p + b_3 \Delta q + b_4 \Delta u \\ + b_5 \Delta \nu + b_6 \Delta \Lambda_0 + b_7 \Delta P_0 + b_8 \Delta Q_0 \\ + b_9 \Delta U_0 + b_{10} \Delta V_0 - \Delta D$$

where

$$\begin{aligned} a_1 &= \gamma_1 k_1 (\varphi_1 \sin \lambda + \varphi_2 \cos \lambda), & b_1 &= k_2 a_1, \\ a_2 &= -\gamma_1 k_1 k_2 \rho \sin \lambda, & b_2 &= -\frac{1}{k_2} a_2, \\ a_3 &= \gamma_1 k_1 k_2 \rho \cos \lambda, & b_3 &= -\frac{1}{k_2} a_3, \\ a_4 &= \frac{\gamma_1 k_1}{2} [\varphi_1 (3 - \cos 2\lambda) + \varphi_2 \sin 2\lambda], & b_4 &= k_2 a_4, \\ a_5 &= -\frac{\gamma_1 k_1}{2} [\varphi_2 (3 + \cos 2\lambda) + \varphi_1 \sin 2\lambda], & b_5 &= k_2 a_5, \\ a_6 &= \gamma_2 k_1 (\varphi_1 \sin \lambda_0 + \varphi_2 \cos \lambda_0), & b_6 &= k_2 a_6, \\ a_7 &= -\gamma_2 k_1 k_2 \rho \sin \lambda_0, & b_7 &= -\frac{1}{k_2} a_7, \\ a_8 &= \gamma_2 k_1 k_2 \rho \cos \lambda_0, & b_8 &= -\frac{1}{k_2} a_8, \\ a_9 &= \frac{\gamma_2 k_1}{2} [\varphi_1 (3 - \cos 2\lambda_0) + \varphi_2 \sin 2\lambda_0], & b_9 &= k_2 a_9, \\ a_{10} &= -\frac{\gamma_2 k_1}{2} [\varphi_2 (3 + \cos 2\lambda_0) + \varphi_1 \sin 2\lambda_0], & b_{10} &= k_2 a_{10}, \end{aligned}$$

and

$$\begin{aligned} \gamma_1 &= \min \left(\frac{a}{a_0}, 1 \right), & \gamma_2 &= \min \left(\frac{a_0}{a}, 1 \right), \\ \varphi_1 &= \gamma_1 \sin \lambda + \gamma_2 \sin \lambda_0, & \varphi_2 &= \gamma_1 \cos \lambda + \gamma_2 \cos \lambda_0, \\ \rho^2 &= \varphi_1^2 + \varphi_2^2, \\ k_1 &= \frac{\cos \varepsilon}{\rho \sqrt{\rho^2 - \varphi_1^2 \sin^2 \varepsilon}}, & k_2 &= \frac{1}{\rho} \varphi_2 \tan \varepsilon \end{aligned}$$

where λ is the mean longitude of a planet in its orbit, λ_0 is heliocentric mean longitude of the Sun, a and a_0 the semi-axes of the planet's and Earth's orbits, ε the obliquity of the ecliptic. The elements of Lagrange-Laplace, used as unknowns in the conditional equations, related with corrections to the Ke-

plerian elements of Earth's and planet's orbits are as follows (Sveshnikov 1985):

$$\begin{aligned} \Delta \varepsilon &= \Delta P_0 & \Delta I &= \Delta p \\ \Delta E &= -\Delta Q_0 / \sin \varepsilon & \sin i \Delta \Omega &= (\Delta q - \Delta Q_0) / \sin \varepsilon \\ \Delta L_0 &= \Delta \Lambda_0 - \Delta Q_0 \operatorname{ctg} \varepsilon & \Delta L &= \Delta \lambda^0 - \Delta Q_0 \operatorname{ctg} \varepsilon \\ \Delta e_0 &= \Delta V_0 \sin \pi_0 & \Delta e &= \Delta \nu \sin \pi + \Delta u \cos \pi \\ &+ \Delta U_0 \cos \pi_0 & & \\ e_0 \Delta \pi_0 &= \Delta V_0 \cos \pi_0 & e \Delta \pi &= \Delta \nu \cos \pi - \Delta u \sin \pi \\ &- \Delta U_0 \sin \pi_0 & & \end{aligned}$$

where the first 5 corrections and ΔD are defined in Sect. 6, and ΔI , $\Delta \Omega$, ΔL , Δe , $\Delta \pi$ are corrections to the inclination of the planet's orbit, to the longitude of the ascending node, to the origin of the mean longitude, eccentricity and the longitude of perihelion of a planet respectively.

Using these relationships, we can re-write the conditional equations so that corrections to the Keplerian elements be unknowns to solved for:

$$\begin{aligned} (O - C)_\alpha \cos \delta &= a_1 \Delta L + a_2 \Delta I + a_3 \sin \varepsilon (\sin I \Delta \Omega) \\ &+ c_1 \Delta e + c_2 (e \Delta \pi) + a_6 \Delta L_0 + a_7 \Delta e \\ &+ c_3 \Delta e_0 + c_4 (e_0 \Delta \pi_0) - \Delta E \cos \delta \end{aligned} \quad (A1)$$

$$\begin{aligned} (O - C)_\delta &= b_1 \Delta L + b_2 \Delta I + b_3 \sin \varepsilon (\sin I \Delta \Omega) \\ &+ d_1 \Delta e + d_2 (e \Delta \pi) + b_6 \Delta L_0 + b_7 \Delta e \\ &+ d_3 \Delta e_0 + d_4 (e_0 \Delta \pi_0) - \Delta D \end{aligned}$$

where

$$\begin{aligned} c_1 &= a_4 \cos \pi + a_5 \sin \pi, & d_1 &= b_4 \cos \pi + b_5 \sin \pi, \\ c_2 &= -a_4 \sin \pi + a_5 \cos \pi, & d_2 &= -b_4 \sin \pi + b_5 \cos \pi, \\ c_3 &= a_9 \cos \pi_0 + a_{10} \sin \pi_0, & d_3 &= b_9 \cos \pi_0 + b_{10} \sin \pi_0, \\ c_4 &= -a_9 \sin \pi_0 + a_{10} \cos \pi_0, & d_4 &= -b_9 \sin \pi_0 + b_{10} \cos \pi_0. \end{aligned}$$

It should be noted that the coefficient of ΔE becomes $-\cos \delta$ in right ascension and 0 in declination when the terms containing ΔQ_0 , $\Delta \Lambda_0$, Δq , $\Delta \lambda^0$ are reduced to ΔE , ΔL_0 , $\sin I \Delta \Omega$, ΔL , see (Sveshnikov 1972).

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