Second-Order Doppler-Effect Experiment.

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Contrary to the usual textbook consensus, there is no satisfactory experimental evidence on the second-order Doppler effects predicted in the special theory of relativity. A simple inexpensive Doppler-effect experiment is suggested that avoids the severe and unmet tolerances of past investigations.

A critical appraisal of past experiments of the Ives-Stilwell type on the Doppler effect on visible light emitted by a glowing high-speed beam of hydrogen atoms has lead to the rather surprising conclusion that the results are grossly inaccurate (1). This is due to the formidable, if not almost unsormountable, technical difficulties inherent in these experiments.

The wavelength measurements were from high hundreds to tens of thousands of times too gross to be consistent with the attained speed of the excited atoms, and the stated experimental errors in the wavelength are unacceptably huge. The *vector* velocity distribution of the excited hydrogen atoms within the hydrogen beam cross-section is by no means well constrained in the direction of the beam. The *vector* velocity distribution over a range of almost 90° to the beam direction results in a chaotic predominant first-order Doppler broadening of the spectral lines to about 0.6 Å in which the doublet structure separation of about 0.1 Å—comparable to the sought for second-order Doppler effect—was not discernable even with microdensitometry.

The small angle between the line of sight and the gross beam direction was known only to an accuracy of $\pm 33\%$. The general lack of necessary precision together with the first-order asymmetric Doppler line broadening renders pointing a meaningful position in the confused spectral lines as a hopelessly impossible task for the detection of second-order Doppler effects.

Certain Mössbauer-effect experiments on the *uncollimated* emission of gamma-rays from an emitter to their *uncollimated* reception at an absorber, both of which were mounted to different positions on a rotating disk, have been *incorrectly* described in terms of the Einstein-Doppler formula *for relative uniform translation* (2). There was, in these experiments, absolutely *no relative motion*, rectilinear or otherwise, between the emitter and the absorber subjected to *accelerated* circular motion due to the rotation of the disk. It is also of interest to note that the observed resonant absorption ef-

(1) W. KANTOR: *Spectr. Lett.*, **4**, 61, 294 (1971).
(2) F. BENNEWITZ: *Phys. Lett.*, **19**, 282 (1965).

fects did *not* involve any measurement of frequency or wavelength so that Doppler effect, while a superficially suggestive inference, was not in fact an experimentally observed phenomenon. The observed absorption cross-section effects are explicable for a relative speed, rather than an absolute speed, of the gamma-ray photons in the same manner that they are explicable for any resonance effect that is dependent on the variable speed of the particles incident on a resonant nucleus.

The Compton collision of photons and free stationary electrons are not well appreciated as also representative of Doppler effect. The experimental evidence on Compton effect and also inverse Compton recoil of photons from free energetic (5.5 GeV) electrons do not provide satisfactory evidence for second-order Doppler effect in the gamma-ray region. The comparison of prediction with experiment for Compton effect is not satisfactory, while the broad *qualitative* observations on inverse Compton effect are not definitive of uniquely conclusive results [3].

The absence of *any* valid second-order Doppler-effect observations presents a need for *direct* experimental confirmation of second-order Doppler effect as an urgent and unsuspected opportunity that cannot be denied. A simple experiment that does not impose excessively strict constraints can be achieved with a source and a receiver at relative rest. The moving element is a plane parallel thin plate of a transparent dispersive medium that translates with the vector velocity $\pm\boldsymbol{v}$ in the direction of a well-collimated laser beam that is incident reasonably normal to the plate.

According to the Einstein-Doppler formula the Doppler effect at the first relatively moving interface encountered by the laser beam is

$$\nu' = \nu\gamma(1 \mp \beta\cos\varphi) = \nu/\gamma(1 \pm \beta\cos\theta)\,, \tag{1}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. The vectors $\boldsymbol{v}$, of the moving plate, and $\boldsymbol{c}$, of the light beam, subtend the angle φ in the reference system in which the light *source is at rest*; $|\boldsymbol{v}\cdot\boldsymbol{c}|/c^2 = \beta\cos\varphi$. The negative sign before $\beta\cos\varphi$ in (1) signifies relatively receeding motion between the *stationary* source and the first uniformly *moving* plate surface, the positive sign denotes relative approach. In a reference system in which the plate is at rest and the *source is in motion* the subtended angle is θ and $|\boldsymbol{v}\cdot\boldsymbol{c}|/c^2 = \beta\cos\theta$. The positive sign before $\beta\cos\theta$ in (1) signifies relative recession of the *moving* source, the negative sign denotes approach. It follows from the last equality in (1) that

$$\cos\theta = (\cos\varphi \mp \beta)/(1 \mp \beta\cos\varphi)\,. \tag{1a}$$

Relative to the reference system in which the plate is at rest the subtended angle at reception is θ at the first surface. This is clearly equal to the same angle θ on re-emission at the second stationary interface so that the Einstein-Doppler formula for the Doppler effect at the relatively moving receiver is simply

$$\nu'' = \nu'\gamma(1 \pm \beta\cos\theta) = \nu'/\gamma(1 \mp \beta\cos\varphi)\,, \tag{2}$$

where the last equality follows from the first together with (1*a*). The positive sign before $\beta\cos\theta$ signifies relative approach of the *moving receiver*—plate stationary,—the negative sign relative recession. The negative sign before $\beta\cos\varphi$ denotes relative ap-

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(3) W. KANTOR: *Spectr. Lett.*, **4**, 59 (1971).

proach of the second re-emitting moving plate surface—receiver stationary,—the positive sign signifies relative recession. It follows from (1) and (2) that $\nu'' \equiv \nu$ independently of the angle subtended between $\boldsymbol{v}$ and $\boldsymbol{c}$.

According to *etherless* classical photon kinematics the Doppler effect corresponding to the relativistic expression (1) is

$$\nu' = \nu(1 \mp \beta \cos\varphi) . \tag{3}$$

Relative to the system in which the plate is stationary the speed of the light c' incident on the plate is readily deduced by reference to Fig. 1 to be $c' = c\sqrt{1 \mp 2\beta\cos\varphi + \beta^2}$.

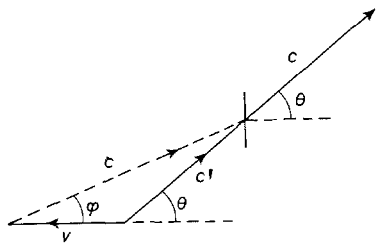

Fig. 1. – « Stationary » plate.

It is also obvious that $c'\cos\theta = c\cos\varphi \mp v$ so that

$$\cos\theta = (\cos\varphi \mp \beta)/\sqrt{1 \mp 2\beta\cos\varphi + \beta^2} , \tag{4}$$

in interesting contrast to (1*a*). The speed of the light re-emitted by the second surface of the stationary plate is changed from the incident value c' to c. The Doppler effect received at the relatively moving receiver—plate stationary—is expressed as

$$\nu'' = \nu'(1 \pm \beta\cos\theta) . \tag{5a}$$

It follows from (3), (4) and (5*a*) to second-order accuracy that

$$\nu'' = \nu(1 - \beta^2) , \tag{5b}$$

so that

$$\Delta\nu = \nu v^2/c^2 . \tag{5c}$$

The classical nonnull result, to second-order accuracy, is also independent of any subtended angles.

The suggested experiment is *not* critically sensitive to angle-dependent first-order Doppler-effect terms which was such a severe handicap in other Doppler-effect experiments. The experiment permits a clear distinction—that is not possible in first-order Doppler-effect experiments—between the exact relativistically predicted Doppler-effect null result and the classically predicted second-order Doppler effect $\nu v^2/c^2$.

As a practical matter the high-speed moving plate might be a light-weight small thin film or flake of glass. Convection effects, while no problem in any case, would, nevertheless, be negligible with a thin film. The laser beam could be bifurcated by a beam splitter so that there would be two beams of equal intensity whereby one incident beam traverses the moving glass flake, while the other traverses an identical stationary glass flake. Combining the two beams afterward at the stationary receiver should permit the detection of a beat frequency difference $\nu v^2/c^2$ classically expected or a null result relativistically predicted.

A linear speed $v = 6000$ cm/s is not a difficult achievement so that with a light frequency $\nu = 5 \cdot 10^{14}$ Hz the classically expected beat frequency would be $\Delta\nu = 20$ Hz. The discernment of one cycle of the beat frequency would require that the glass flake move a distance of at least $v/\Delta\nu = 300$ cm. This requirement could be reduced by reflecting the laser beam back and forth through the moving glass film ten or more times, depending on the diameter of the laser beam and the transverse dimension of the glass film. Frequency doubling of the beat frequency occurs for each reflection. Thousands of reflections, if possible, offer the convenient possibility of smaller v and also smaller traversed distance.

Conceptually, the experiment could be inexpensively realized by mounting a glass flake so that it protrudes from a flexible belt that is friction-driven from the rim of a disk 12 cm in radius rotating at 4 800 r.p.m. The endless belt would pass over a small-diameter idler bearing located the necessary distance in the plane of the disk from the axis of the disk. The laser beam would traverse the glass along a path just adjacent and parallel to the taut side of the belt moving linearly at a speed of 6000 cm/s.