Radial velocity and intensity of light, by *W. de Sitter.*

In *Zeitschrift für Physik*, XXI, 6, p. 333 (1924) Prof. M. LA ROSA has pointed out a remarkable consequence of the theory of RITZ regarding the propagation of light, which seems not to have been noticed before. If this theory, according to which the velocity of the source is added to the velocity of light, is true, then the light emitted by a star, whose motion relatively to the observer is perodic, during equal intervals of time, will reach the observer during unequal intervals, the inequality increasing with the distance of the star. Consequently the star will appear to the observer to be variable, even if its emitted light has a constant intensity. This, of course, is entirely correct, but, contrary to the opinion of Mr. LA ROSA, it does not afford an argument in favour of RITZ's theory, but rather against it.

If the waves emitted by the source during the interval of time dt reach the observer during the interval $dt' = dt\,(1 + q)$, then the observer will, on the one hand, ascribe to the star an intensity $i = i_0/(1 + q)$, if the true intensity be i_0, and on the other hand he will, according to Doppler's principle, ascribe to it a radial velocity $v = cq$ *) (positive for recession), if c be the velocity of light from a source without relative radial motion relatively to the observer. Both effects depend on *the same* factor q. Consequently we have, neglecting the square of q:

$$-\frac{\Delta i}{i_0} = q = \frac{v}{c},$$

or, since one magnitude corresponds to 0.4 in the logarithm of the intensity, for small values of q:

$$\Delta\, m = 1{\cdot}086\, q,$$

and consequently

$$v = 277000\, \Delta\, m, \tag{1}$$

if the velocity v is expressed in kilometers per second. Thus, if the variability of the star were due to its motion, then to a variation of a few tenths of a magnitude would correspond by (1) a change of apparent wavelength which would by Doppler's principle be interpreted as a velocity of the same order as the velocity of light.

For large values of q the above formulas are no longer correct, but we have

$$\Delta\, m = 2{\cdot}5 \log\,(1 + q),$$

and we find the following corresponding values of $\Delta\, m$ and v:

$\Delta\, m$	v classical theory $\Delta\, m +$	$\Delta\, m -$	v relativity theory	approximate formula (1)
$\pm$ 0.1 m	+ 29 000	− 26 000	$\pm$ 28 000	$\pm$ 28 000
$\pm$ 0.2	+ 61 000	− 50 000	$\pm$ 55 000	$\pm$ 55 000
$\pm$ 0.5	+ 176 000	− 114 000	$\pm$ 129 000	$\pm$ 139 000
$\pm$ 1.0	+ 434 000	− 181 000	$\pm$ 218 000	$\pm$ 277 000

The velocities are thus of the same order as by the approximate formula (1), though those found according to the theory of relativity can, of course, never exceed the velocity of light.

On the other hand the variations of magnitude, which undoubtedly accompany any real radial velocity according to the principle pointed out by LA ROSA, are so small that they are entirely unobservable. To a velocity of 300 km. sec^{-1}, which is about the largest occurring amongst double stars, would correspond a change of brightness of about 0·001 magnitude.

Take as an example a star moving with uniform angular velocity n in a circle with radius a in a plane passing through the observer. The star's distance from the observer then is

$$\Delta' = \Delta - a \sin nt,$$

and the radial component of its velocity is

$$v = an \cos nt.$$

Now let the velocity of the light emitted by the star towards the observer be

$$c' = c + \varkappa\, v,$$

where $\varkappa = 1$ in RITZ's theory and $\varkappa = 0$ in the usual theory. Then the light which leaves the star at the time t reaches the observer at the time $t' = t + \Delta'/c'$ or

$$t' = t + \frac{1}{c}(\Delta - a \sin nt)\,\left(1 + \varkappa\frac{an}{c}\cos nt\right)^{-1}.$$

The second factor can be developed, and we can neglect the square of an/c. Then differentiating we find

$$\frac{dt'}{dt} = 1 - \frac{an}{c}\cos nt + \varkappa\frac{an^2\Delta}{c^2}\sin nt. \tag{2}$$

Consequently in the formula $dt' = dt\,(1 + q)$ we have

$$\begin{aligned} q &= -\frac{an}{c}\left[\cos nt - \varkappa\frac{n\Delta}{c}\sin nt\right] \\ &= -K\cos(nt + \varepsilon), \end{aligned} \tag{3}$$

*) This is the formula according to the classical theory for a moving source, the observer being at rest. The theory of relativity of course gives $\sqrt{(c+v)/(c-v)} = 1 + q$, v being the relative velocity.

where
$$\tan\varepsilon = \varkappa \frac{n\Delta}{c},$$

$$K = \frac{an}{c} \sec\varepsilon.$$

The second term in (2) and (3) containing Δ may become very large for distant stars, unless $\varkappa$ is very small. Prof. LA ROSA neglects the first term, which, of course, for large values of Δ is much smaller than the second one if $\varkappa = 1$. But, if the second term is preponderating in the effect on the magnitude, it is also preponderating for the Doppler effect, and will give rise to enormous radial velocities. Since these are not observed, the magnitude effect can only be very small, and we must conclude that the second term does not preponderate.

ZURHELLEN has pointed out in 1914 (*A. N.* **198**, 4927, p. 1) that the angle ε, which occurs in (3), can be actually determined in the case of eclipsing variables with known spectroscopic orbits, since it represents the difference of the phase as determined from the radial-velocity curve and from the light-curve of the eclipse. He discusses 7 stars, for all of which the upper limit of a possible difference of phase is found to be smaller than about half an hour. Then from two stars with known parallaxes (Algol and β Aurigae) he concludes that $\varkappa$ cannot exceed one millionth. ZURHELLEN of course does not mention the effect on the magnitude and as a matter of fact for ordinary velocities, such as he was dealing with, it is entirely negligible, as has already been mentioned above.

I may mention in this connection, though it does not affect our present argument, that LA ROSA's opinion, that the only stars for which Kepler's laws have been actually verified are the visual binaries with known orbits, appears to me to be incorrect. In fact, the same laws are involved in the determination of spectroscopic orbits, and are consequently, if the observed velocities are well represented by the computed orbit, confirmed by these observations.