A Theory of Action-at-a-Distance

By G. BURNISTON BROWN
University College, University of London

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Abstract. A theory of non-instantaneous action-at-a-distance and physical relativity is used to develop a force-formula for particles in relative motion. The velocity terms are derived from the forces between electrical circuits, and the acceleration terms from experiments with oscillators. The formula so obtained, when corrected for 'retardation' accounts for electromagnetic induction.

On the hypothesis that the variation of force with motion is the same for gravitational forces, an explanation of inertia is given in terms of the total amount of matter in the universe and its distribution, in quantitative agreement with present estimates of the mean density. A 'red-shift' near large bodies and a perihelion motion of the planets also follows. This unified theory of gravitational and electrical force appears capable of giving a physical explanation of the macroscopic phenomena of physics.

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§ 1. Introduction

It is well known that the idea of action-at-a-distance has usually met with repugnance, and that Newton rejected it. Physicists have preferred emission theories or medium theories, it being generally assumed that 'contact' forces do not require further elucidation. It was never clear, however, that contact was not action-at-a-distance with the distance very small.

An examination of the application of the hypothesis of action-at-a-distance to macroscopic physics is the object of the present paper; it is defined as follows

Action-at-a-distance.

The forces exerted between particles of matter act at a distance, i.e. without the necessity for any intervening matter.

To this is added

Non-instantaneous action.

All particles in the universe are continually interacting, but a particle at time t can only affect another particle with respect to which it is in motion at a distance r at a time $(t+r/c)$ later (where $c \simeq 3 \times 10^{10}$ cm sec^{-1}).

Newton, following Galileo, treated forces as acting independently, so that the force between two particles depended only on the particles themselves and their relative distance apart, and this principle of independence and relativity is extended to include motion as follows:

The principle of physical relativity.†

Every particle acts on every other particle with a force which depends only on the particles themselves, their relative separation and motion, and the constant of interaction c.‡

† So called to distinguish it from the principle of relativity which is concerned with the relations between moving observers' measurements.

‡ For the advantages of referring to c as the constant of interaction rather than as the velocity of anything, see Brown (1941).

These, together with the principle of superposition, are the postulates of this theory of non-instantaneous action-at-a-distance and physical relativity. We now attempt to determine, from information derived from experiment and observation, the extra terms required to extend the macroscopic force-formula to include the effects produced by relative motion, just as Newton, in finding the first term, used the results of Kepler and Tycho Brahe.

§ 2. Velocity Terms

We start with the fact that when two particles are at rest relatively to one another, the macroscopic interactions, gravitational and electrical, are represented by similar force-formulae viz. mm'/r^2 and qq'/r^2 where m, m' are the masses in dynamical units, q, q' are the charges in e.s.u., and r is the distance between them. This similarity suggests the hypothesis that the variation of force with motion is also similar in the two cases.

Proceeding, then, on this hypothesis, we further assume, with Ampère, that all magnetic effects are the result of the motion of charges, and that in neutral conductors carrying steady currents, we have charged particles moving with an effective resultant 'drift' velocity v, and in oscillators we have charges moving with acceleration f as well.

Commencing with the v terms, we take "the most general form of Ampère's law consistent with the experimental facts" (Maxwell 1904). For steady, uniform currents when one, at least, of the circuits is closed, we have for the force in the x direction between two elements ds, ds' carrying currents i and i', distant r apart:

$$F_x = -ii'dsds'[R\cos(rx) + S\cos(xds) + S'\cos(xds')]$$

where

$$R = \frac{1}{r^2}\left(\frac{\partial r}{\partial s}\frac{\partial r}{\partial s'} - 2r\frac{\partial^2 r}{\partial s \partial s'}\right) + r\frac{\partial^2 Q}{\partial s \partial s'}, \qquad S = \frac{\partial Q}{\partial s'}, \qquad S' = -\frac{\partial Q}{\partial s}.$$

The added Q terms give zero when integrated round a complete circuit, and are therefore compatible with Ampère's results. Taking $Q = K/r$ where K is some constant, we can show that

$$F_x = -\frac{ii'dsds'}{r^2}\{[(K+2)\cos(dsds') - 3(K+1)\cos(rds)\cos(rds')]\cos(rx) + K\cos(rds')\cos(xds) + K\cos(rds)\cos(xds')\}.$$

To convert this expression for F_x into one involving charges q, q' and velocities v and v', we make use of the experimentally determined relation $qv/c = ids$ (Rowland and Hutchinson 1889). Thus for the force we write

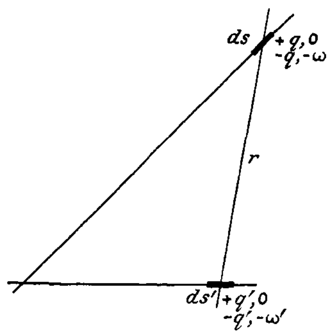
$$E_x = \frac{-qq'}{r^2c^2}\{[(K+2)vv'\cos(dsds') - 3(K+1)v_r v_r']\cos(rx) + Kv_x'v_r + Kv_x v_r'\} \qquad \ldots\ldots(1)$$

since v is along ds and v' along ds'.

With physical relativity, only the relative velocity affects the force, and so, noting that dimensionally the terms are [velocity^2], we shall have terms in v^2, v_r^2, and $v_x v_r$. To find the coefficients of these terms we must calculate the resultant force of one neutral element of current on another neutral element.

§ 3. Force between Linear Circuits at Rest

We now have to consider the neutral current elements ids and $i'ds'$, as consisting of positive charges $+q$, $+q'$ at rest, and negative charges $-q$, $-q'$ in motion (see figure). Let the velocity of the negative charge in ds be $-\omega$ and in ds', $-\omega'$.



Taking axes in the conductor element ds, the force of the element ds' on ds will be the resultant of four forces: the forces on $+q$ and $-q$ (velocity $-\omega$) at ds, due to $+q'$ (velocity zero) and $-q'$ (velocity $-\omega'$) at ds'.

The terms in v^2 are

$$\begin{aligned}\sum qq'v^2 &= -qq'(\omega_x')^2 - qq'(\omega_x)^2 + qq'(\omega_x - \omega_x')^2 + \ldots + \ldots \\ &= -2qq'\sum \omega_x\omega_x' = -2c^2 ii'dsds' \cos(dsds').\end{aligned}$$

Similarly

$$\sum qq'v_r^2 = -2c^2 ii'dsds' \cos(rds)\cos(rds')$$

$$\sum qq'v_x v_r = -c^2 ii'dsds'[\cos(xds)\cos(rds') + \cos(rds)\cos(xds')].$$

Comparing these terms with the most general expression of Ampère's formula (1), we see that if we multiply the v^2 terms by $-(K+2)/2c^2$, the v_r^2 terms by $-3(K+1)/2c^2$, and the $v_x v_r$ terms by $-K/c^2$, we shall obtain agreement. Thus we have for the velocity terms:

$$E_x = \frac{qq'}{r^2}\left\{\left[\frac{(K+2)}{2}\frac{v^2}{c^2} - \frac{3(K+1)}{2}\frac{v_r^2}{c^2}\right]\cos(rx) + \frac{Kv_x v_r}{c^2}\right\}. \qquad \ldots\ldots(2)$$

§ 4. Acceleration Terms

In order, next, to derive the acceleration terms, the dipole oscillator suggests itself as perhaps the simplest case to take; and if we consider the interaction effects produced at distances large compared with the amplitude of the vibration of the current, then using polars, we can treat r and θ as the same for all parts of the oscillator, and the only effect of 'retardation' (i.e. the finite interaction constant c) will be in the phase.

We shall assume that experiment shows (Hertz 1900, Ratcliffe 1931–2, McPetrie, Perry and Ford 1945) that a dipole oscillator, at large distances, only produces electric force at right angles to the radius vector r in a plane containing r and the direction of oscillation, of amount $E = (qap^2 \sin\theta/rc^2)\cos p(t - r/c)$ where a is the amplitude of the oscillation of a charge q, p is the pulsatance, and θ the angle between r and the direction of oscillation. Neglecting the phase retardation and remembering that f (the acceleration) is $-ap^2 \cos pt$, we find that $E_x = (qq'/rc^2)[f_r \cos(rx) - f_x]$ satisfies the conditions.

An oscillating dipole is not, of course, a neutral conductor: there are free fluctuating charges at its ends, but the force due to these decreases as $1/r^3$ and so rapidly becomes negligible compared with $1/r$. The velocity terms compared with the acceleration terms are of the order v^2/rf or $a^2p^2/ap^2r = a/r$ and are therefore negligible.

Having obtained the acceleration terms from experiments in which r and $\cos(rx)$ are sensibly the same for all parts of the circuit, so that retardation effects due to changes in them are not appreciable, we must now consider the case in which this is not so, e.g. ordinary induction with linear circuits in the laboratory.

Now retardation makes the effect at time t of a particle P on a particle Q (with respect to which it is moving), depend on the position and motion of P at a time $(t - R/c)$ where R was the distance separating P and Q at that time. We can use Taylor's theorem in the form

$$x\left(t - \frac{R}{c}\right) = x(t) - \frac{R}{c} v_x(t) + \frac{R^2}{2c^2} f_x(t) + \dots$$

$$v_x\left(t - \frac{R}{c}\right) = v_x(t) - \frac{R}{c} f_x(t) + \dots$$

to express the 'retarded' function $\cos(Rx)/R$ in terms of the instantaneous values $\cos(rx)/r$, provided the expansions are rapidly convergent. Assuming that this condition is satisfied in practice, we can show that for circuits in which r and $\cos(rx)$ vary, and therefore cause varying retardation, the acceleration terms are

$$-\frac{qq'}{2rc^2}[f_r \cos(rx) + f_x]. \qquad \dots\dots(3)$$

§ 5. Electromagnetic Induction

When we consider the inductive effect in an element of a circuit due to another element, in the same or another circuit, we have to find the force tending to separate the positive and negative charges in the element in the direction of the inducing element, and sum this up for the whole circuit. From this force, when it refers to unit charge, can be found the induced electromotive force.

It is not difficult to show that the velocity terms and the acceleration terms derived above, both, independently, yield e.m.f. $= -dN/dt$ for the induced electromotive force.

§ 6. The Law of Gravitation and the Motion of Mercury

We have now obtained from electrodynamic experiments the extra terms involving the velocity (equation (2)) and the acceleration (equation (3)) which have to be added to the electrostatic force-formula in order to account for the facts of electromagnetism. If we proceed by making the hypothesis, already mentioned, that gravitational force varies in the same way, we can obtain the gravitational force between two masses m and m' (dynamical units) by merely substituting m and m' for e and e', and changing the sign, and we then have

$$F_x = -\frac{mm'}{r^2}\left\{\left[1 + \frac{(K+2)}{2}\frac{v^2}{c^2} - \frac{3(K+1)}{2}\frac{v_r^2}{c^2} - \frac{rf_r}{2c^2}\right]\cos(rx) + \frac{Kv_x v_r}{c^2} - \frac{rf_x}{2c^2}\right\}.$$

Now, if a case were known where there was a discrepancy between the Newtonian formula and observation, and this discrepancy could be measured with a fair degree of accuracy, we might be able to determine the value of the constant K. The movement of the perihelion of Mercury provides such an instance. In this planetary case the acceleration is negligible except along the radius vector, and this is given very closely by $-GM/r^2$ where M = mass of the Sun in grammes. If this value is substituted in the acceleration terms, the force formula leads to a rotation of the perihelion per revolution of

$$\frac{(3-K)\pi GM}{c^2 a(1-e^2)}$$

where e is the eccentricity and a is the semi-major axis of the orbit. If we take $K=-3$ we get the formula first obtained by Gerber in 1898, and by Einstein in 1916. Ritz obtained a similar formula in 1908 but neglected the acceleration terms. We are not, of course, compelled to choose K a whole number, but the value -3 gives agreement with experiment within the limits of accuracy:

Observed value for Mercury $42''\cdot56 \pm 0\cdot94$ per century (Clemence 1947)

Calculated ($K=-3$) $43''\cdot03 \pm 0\cdot03$

Thus the force-formulae become

$$E_x = \frac{qq'}{r^2}\left\{\left[1-\frac{1}{2}\frac{v^2}{c^2}+\frac{3v_r^2}{c^2}-\frac{rf_r}{2c^2}\right]\cos(rx)-\frac{3v_x v_r}{c^2}-\frac{rf_x}{2c^2}\right\} \qquad \ldots\ldots(4)$$

$$F_x = -\frac{mm'}{r^2}\left\{\left[1-\frac{1}{2}\frac{v^2}{c^2}+\frac{3v_r^2}{c^2}-\frac{rf_r}{2c^2}\right]\cos(rx)-\frac{3v_x v_r}{c^2}-\frac{rf_x}{2c^2}\right\}. \qquad \ldots\ldots(5)$$

§ 7. Origin of Inertial Forces

Having obtained a formula for the interaction of uncharged particles of matter which takes account of relative velocity and acceleration, we can inquire into the problem of *inertia*. If this is not due to movement with respect to 'absolute space', it ought to be due to surrounding matter, as suggested by Bishop Berkeley when criticizing Newton, and later by Mach. Now the evidence of astronomical observation at the present time is that the matter of the universe is distributed more or less uniformly, and to about the same distance in all directions. We must therefore consider the force on a moving body at the centre of a spherical distribution of matter of uniform density ρ (dynamical units) and radius R. Using the postulate of physical relativity, we can take our particle of mass m to be at the centre of coordinates, and the universe moving in the opposite direction. On calculating the force by equation (5) we find that for a steady velocity the force of the universe on m is zero, but for an acceleration f there is an opposing force equal to $-(4/3)(\pi m\rho R^2/c^2)\times f$. If we take this to be the force of inertia and write m_1 for the inertial mass, we shall have

$$F = m_1 f = \frac{4}{3}\frac{\pi\rho R^2}{c^2} mf.$$

Thus the ratio of the attractive mass to the inertial mass of a body which we know to be $\sqrt{G}$ should be given by $3c^2/4\pi\rho R^2$ or

$$G = \frac{9c^4}{16\pi^2\rho^2 R^4}. \qquad \ldots\ldots(6)$$

Taking $G = 6{\cdot}7 \times 10^{-8}$ and $R = 2 \times 10^{27}$ cm we can calculate the mean density of matter in the universe from equation (6) which yields 10^{-27} g cm^{-3}, a result which agrees with present estimates (Zwicky 1952).

§ 8. Mass and Radius of a Spherical Universe and Red-Shift

With ρ in usual units, equation (6) becomes $GM/c^2 = R$, where M = mass of the universe. For the 'Einstein universe' $GM/c^2 = 1{\cdot}57R$, (Eddington 1949) and Whitrow (1950), as a result of a "concise argument partly based on Newtonian mechanics" gets $GM/c^2 = 1{\cdot}67R$.

Equation (6) also allows the increased inertial effect near a spherical body of mass m_s to be calculated, and the effect of this on the frequency of a rotating or oscillating particle can be shown to be a 'red-shift' by a factor $1 - \frac{1}{2}\rho_s r_s{}^2/\rho R^2$ where ρ_s and r_s are the density and radius of the body. In the case of the Sun this is of the order of one part in a million. As is known, a shift of this order is present on the Sun's limb.

§ 9. The Force on a Moving Electron

The velocity terms in the force-formula have been derived from experiments on the force between current circuits, where the velocities of the moving charges concerned are, as far as we know, small compared with c. It is interesting to see how far the formula holds for moving free electrons. If we take, first, the case where the electron moves with velocity v at right angles to an electric field, as it does in experiments of the Bucherer type, we can show that the force on it is no longer $4\pi\sigma e$ (where the symbols have the usual meanings) but $4\pi\sigma e(1 + \frac{1}{2}v^2/c^2)$.

Dividing by m_i, the acceleration could be written (if v^2/c^2 is small)

$$f = \frac{4\pi\sigma e}{m_i(1 - v^2/c^2)^{1/2}}$$

so that if the variation of electric force with velocity is treated as a change in m_i, this would agree with $m = m_0(1 - v^2/c^2)^{-1/2}$. Unfortunately the accuracy of this type of experiment is not sufficient (cf. Zahn and Spees 1938) to decide whether the force-formula is correct, or whether it requires the addition of higher powers of v^2/c^2. The case where the electron moves in the direction of the field, as in accelerators, is difficult to deal with, owing to lack of information on the rapidly varying non-uniform fields employed, but the formula indicates a limiting velocity. The force of a neutral current (or magnet) on a moving charged particle is correctly given by the formula.

§ 10. Optical Effects produced by Motion

Action-at-a-distance avoids the need for an ether, and therefore for 'waves'. Particles interact directly with one another, but the presence of other particles in the neighbourhood may, by superposition, cause a change of phase, which we are accustomed to look upon as a 'change in the velocity of light' (cf. Jenkins and White 1951) although, in all interaction, there is no change in the constant c. Bearing this in mind it is not difficult to show that the Doppler effect, aberration, and phenomena in moving bodies, can be adequately explained in terms of action-at-a-distance.

The Michelson–Morley and Kennedy–Thorndike experiments yield results which would be expected, since without an ether, movement is only with respect to the celestial bodies, and uniform motion with respect to these produces, a we have seen, no forces, and consequently no changes. The experiments o Ives and Stilwell, Michelson and Gale, Sagnac, and Dufour and Prunier, requir a more detailed criticism which it is hoped to publish at a later date, when i will be shown that they do not invalidate the present theory.

§ 11. Conclusion

Experiment shows that electrical oscillations produce a very small force in a radial direction (Cullen 1952) (radiation pressure) so that the force-formula requires a small extra term, and phenomena occurring when $v \to c$ may also need extra terms. When more accurate experimental results become available it will be possible to decide what these terms should be. The theory o non-instantaneous action-at-a-distance and physical relativity seems capable o giving a causal explanation of the phenomena of macroscopic physics in terms o matter, force and motion; and (with the above reservations) the formula found gives quantitative agreement with experiment.

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