THE BARR EFFECT: A STATISTICAL STUDY

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The reality of the 'Barr effect' is established by using nonparametric statistical tests on the DAO *Eighth Catalogue* sample. For orbits of reasonable quality, it is present only for orbital periods $\lesssim 3$ days, with a preferred direction for longitudes of periastron of $\omega \simeq 100°$. These characteristics are qualitatively consistent with measured radial velocities being biassed by gas streams. The Barr effect is not detectable in Griffin's 'Spectroscopic Binary Orbits' series.

Samples of eccentric, short-period spectroscopic-binary orbits tend to show a nonuniform distribution of longitudes of periastron, ω. This 'Barr effect' was first demonstrated by the eponymous J. Miller Barr[1], a Canadian amateur astronomer. His initial report was not warmly received by contemporary professionals[2,3], in hindsight with some justification, not least because several of his 'orbits' were parameterizations of radial-velocity variations actually associated with pulsations. Nonetheless, by the time of Struve's review[4] the Barr effect was widely accepted as being genuine, and it was (and is) attributed to the distortion of radial-velocity curves by gas streams.

Recent discussions have illustrated the Barr effect by means of a 'Barr chart'; that is, by plotting a probability distribution function (PDF) in polar coördinates (*e.g.*, refs. 5, 6). From a qualitative point of view this is an effective way of presenting the data (although, as Griffin[6] remarks, it can be misleading because of the subjective tendency to interpret such a diagram as weighted by area, rather than by radius). However, for quantitative purposes a 'Barr chart' is less satisfactory, because arbitrary choices have to be made concerning bin boundaries. This renders statistical studies difficult; indeed, there appears to be no formal demonstration of the statistical significance of the Barr effect in the literature. The principal purpose of the present note is to consider the statistics of the Barr effect.

As with all finite samples drawn from continuous distributions, the distribution of longitudes of periastron is more objectively presented in the form of a cumulative distribution function (CDF) than as a PDF. Figure 1 shows the CDFs for 920 binaries having eccentric orbits listed in the *Eighth Catalogue*[7], for two subsets of those data, and for 84 binaries from Griffin's series[6].

It is relatively straightforward to investigate the CDF with rigorous, nonparametric statistical tests. In the present case, we wish to test the null hypothesis that the distribution of longitudes of periastron in the parent population is uniform. At first sight, the Kolmogorov-Smirnov (K-S) test is the obvious one to apply (*e.g.*, ref. 8); here the test statistic is D_{KS}, the maximum difference between the hypothesized and observed cumulative distributions. However, in the context of the Barr effect, this test statistic, and the associated probability level P_{KS}, have the undesirable property of not being invariant with choice of origin. For example, taking all *a*-, *b*-, and *c*-quality orbits for systems with

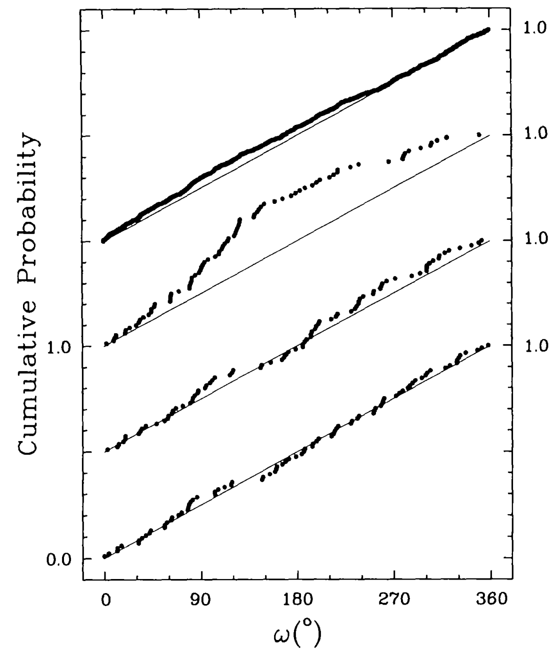

FIG. 1

Cumulative distribution functions for longitudes of periastron. The samples are, from top to bottom, (i) the 920 orbits from the *Catalogue*[7]; (ii) the 80 class *a–c Catalogue* orbits with P_{orb} < 3 days; (iii) the 83 class *a–c* orbits with P_{orb} > 1000 days; and (iv) the 84 eccentric-orbit systems reviewed by Griffin[6]. The diagonal lines show, for each sample, the CDF for parent populations uniformly distributed in ω.

orbital periods $P_{orb} < 3$ days listed in the *Catalogue*[7] gives $D_{KS} = 0{\cdot}26$, $P_{KS} = 0{\cdot}0\%$; adding 90° to all values of ω (and taking modulo 360°) changes these figures to 0·14, 9·5%. In the former case we would conclude that there is strong evidence for a non-uniform distribution of longitudes of periastron, while in the latter we would conclude that the data were consistent with the null hypothesis of a uniform distribution — and yet the data are the same in both cases!

This dependence on choice of origin is implicit in this application of the K-S test, which is more appropriately used when the parent PDF is centrally peaked. The dependence is removed if we choose as a reference not the model CDF, but a line which has the same slope and which gives a mean (O – C) of zero. A suitable test statistic may then be defined — for example, the normalized r.m.s. dispersion about the offset line, σ_D, or (by analogy with the K-S statistic) the maximum deviation of observed points from that line, D'. The probability distribution of the test statistic may be estimated nonparametrically from Monte-Carlo simulations.

Table I summarizes calculations made in this way for various subsets of the *Catalogue* sample. As a test statistic, σ_D is listed; experiments showed D' to be rather inefficient (*i.e.*, to be relatively insensitive to non-uniform distributions). The corresponding probability levels were estimated by comparison with 1000 artificial datasets, each having the same number of points, N, as the observed samples; those points were drawn at random from a uniform distribution in ω. For orbits of quality *a–c* ("reliable" or better, in the judgement of the compilers of the *Catalogue*), the distribution in ω is significantly non-uniform only for $P_{orb} \lesssim 3$ days. For lower-quality orbits (quality indicators *d–i*; "poor" solutions or

TABLE I

Analysis of the Catalogue Sample

P_{orb} (d)	Sample q	N	σ_D	P_σ(%)	R	P_R (%)	$\theta(R)$
0–∞	*abcdei*	920	0·017	1·8	0·071	1·4	41
0–∞	*abc*	598	0·013	28·4	0·049	22·4	74
0–∞	*dei*	322	0·033	0·4	0·135	0·5	18
0–3	*abcdei*	129	0·059	<0·1	0·252	<0·1	76
0–3	*abc*	80	0·079	<0·1	0·341	<0·1	98
0–3	*dei*	49	0·062	4·6	0·246	4·5	22
3–10	*abcdei*	208	0·031	3·9	0·129	3·1	31
3–10	*abc*	133	0·023	50·0	0·082	40·4	20
3–10	*dei*	75	0·051	3·3	0·218	2·1	39
10–30	*abcdei*	148	0·020	63·0	0·064	56·2	94
10–30	*abc*	107	0·026	49·3	0·096	36·6	97
10–30	*dei*	41	0·042	47·3	0·025	97·1	304
30–100	*abcdei*	92	0·050	2·3	0·210	1·5	325
30–100	*abc*	63	0·036	38·4	0·129	37·0	321
30–100	*dei*	29	0·092	0·7	0·393	0·3	328
100–300	*abcdei*	87	0·021	84·3	0·061	71·6	334
100–300	*abc*	61	0·028	73·1	0·071	72·8	248
100–300	*dei*	26	0·063	21·5	0·256	16·1	15
300–1000	*abcdei*	104	0·015	97·7	0·031	90·4	65
300–1000	*abc*	71	0·019	94·6	0·054	80·8	304
300–1000	*dei*	33	0·048	40·2	0·188	30·6	97
1000–∞	*abcdei*	152	0·018	69·0	0·023	92·5	218
1000–∞	*abc*	83	0·026	67·1	0·051	82·2	183
1000–∞	*dei*	69	0·028	66·5	0·035	92·7	309

Notes: q is the quality of the orbital solution given in the *Eighth Catalogue*[7]; σ_D and R are the test statistics, and P_σ, P_R the corresponding probabilities that the null hypothesis of a uniform distribution in ω is not violated. The preferred direction of ω is quantified by θ (R).

worse) the behaviour is, not surprisingly, less systematic, although there is a suggestion of a significant Barr effect extending to $P_{orb} \lesssim 10$ days. Figure 1 illustrates the difference between short- and long-period systems by displaying CDFs for class *a–c* orbits having $P_{orb} >$ 1000 days, for which the sample sizes are nearly identical.

The preferred direction for ω is not straightforward to define on the basis of the CDF, in part because 'clumping' does not occur on a single, unique scale. The largest positive slopes in the CDF do indicate favoured regions, and correspond to the greatest radial offsets in a 'Barr chart'; but although this criterion again does not suffer from the need for arbitrary choices of bin boundaries, nor from the weighting problem noted by Griffin[6], it is nonetheless uncomfortably subjective. A more objective approach is provided by taking the mean normalized rectangular coördinates of the sample of observed longitudes of periastron (*i.e.*, $\bar{x} = N^{-1} \Sigma_N (\sin \omega_i)$, $\bar{y} = N^{-1} \Sigma_N (\cos \omega_i)$). This not only defines a single preferred direction for ω, but also provides the basis for a second statistical test: the Rayleigh test. The test statistic here is $R = \sqrt{(\bar{x}^2 + \bar{y}^2)}$; confidence levels for non-zero R (*i.e.*, for significant clumping) can again be estimated nonparametrically by Monte-Carlo simulations. Results are included in Table I. The Rayleigh test confirms the results of the σ_D statistic, and is essentially equally efficient.

The fact that the Barr effect is present at a statistically significant level only in short-period systems provides strong circumstantial evidence for an origin in gas streaming, which in general is most likely to occur in close systems. For the full sample of *Catalogue* orbits, the distribution is, by the Rayleigh criterion, peaked towards $\omega \simeq 40°$, in agreement with the result found[5] for the sample from the *Sixth Catalogue*[9]. However, the sample of well-determined, short-period orbits peaks at $\omega \simeq 100°$, considerably further round the orbit. Compared to a sinusoidal radial-velocity curve, $\omega \simeq 100°$ corresponds to a shallower, longer 'rising' branch, and a steeper, shorter 'falling' branch.

If we suppose that the centre-of-mass motion of a given star is actually a circular orbit, then an observed radial-velocity curve with $\omega \simeq 100°$ is consistent, in a schematic way, with the effects of gas streaming either onto or (more probably) from that star. A gas stream from the star will be seen in projection against it from somewhat before superior conjunction until about a quarter of an orbit later. The gas stream will have a line-of-sight velocity of approach which is greater than that of the star, and if it produces absorption lines then those lines may 'pull' the measured stellar velocities to greater negative values (with respect to the γ-velocity), producing an apparently steep 'falling' branch. A gas stream from the second (presumed fainter) star would be seen projected against its companion in, roughly, the quarter-orbit *before* superior conjunction of that companion, and would 'pull' the stellar velocities to greater positive velocities — again producing an apparently steep 'falling' branch.

Finally, the series of 'Spectroscopic Binary Orbits' published in this *Magazine* by Griffin represents a unique sample of long-period orbits established with high-quality data. We should not expect the Barr effect to be evident in that sample if it is a result of gas streams. In reviewing the first 100 papers in the series, Griffin[6] noted a "tantalizing" suggestion of a slight excess of orbits with ω in the second octant, but he concluded (though without an explicit statistical test) that the Barr effect is absent in his sample. Monte-Carlo calculations provide a quantitative confirmation of that conclusion; the first 100 'Orbits' include 84 having significant eccentricity, and they yield $\sigma_D = 0{\cdot}019$, $P_\sigma = 92{\cdot}3\%$, and $R = 0{\cdot}041$, $P_R = 86{\cdot}6\%$.

In summary, there is strong statistical evidence for a non-uniform distribution of observed longitudes of periastron in short-period spectroscopic binaries, which is plausibly interpreted in terms of gas-streaming effects. More-detailed consideration of the makeup of the *Catalogue* sample, together with gas-dynamic calculations, would be necessary to confirm this interpretation quantitatively.

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