THE ASTRONOMICAL JOURNAL, 128:878–888, 2004 August

SECULAR TRENDS IN THE MEAN LONGITUDES OF PLANETS DERIVED FROM OPTICAL OBSERVATIONS

YURI B. KOLESNIK
Institute of Astronomy, Russian Academy of Sciences, ulitsa Pyatnitskaya 48, 119017 Moscow, Russia; kolesnik@inasan.rssi.ru

AND

C. JOHAN MASRELIEZ
The EST Foundation, 3209 West Lake Sammamish Parkway, NE, Redmond, WA 98052; jmasreliez@estfound.org
Received 2003 May 14; accepted 2004 April 30

ABSTRACT

About 240,000 worldwide optical observations of the Sun, Mercury, and Venus, accumulated during the entire era of classical astrometry from James Bradley up to the present, are used to analyze the secular longitude variations of the innermost planets. A reduction method relating historical planetary observations to the *Hipparcos* reference frame is presented. Secular trends in the longitudes of the Sun, Mercury, and Venus with respect to the ephemeris DE405 are estimated for the time span 1750–2000.

Key words: astrometry — celestial mechanics — ephemerides — reference systems — solar system: general

1. HISTORICAL BACKGROUND

The transition to new astronomical standards that began in 1984 was almost completed in 1998 with the introduction of a new primary reference frame based on a set of extragalactic objects. Optical positions of stars in the *Hipparcos* Catalogue, and modern numerical ephemerides of planets in the solar system, are now linked to the International Celestial Reference System (ICRS) nonrotating frame. Precession constants are presently derived independently of star catalogs' proper motions by the use of lunar laser ranging (LLR) and VLBI methods. The tidal acceleration of Earth's rotation is now also estimated from LLR. The quasar-based reference system, extended backward in time, and astronomical constants based on modern techniques provide an opportunity to correctly interpret the secular variations of contiguous, strongly correlated, astronomical parameters such as the location of the equinox, the deceleration of Earth's rotation, and the variation of the longitudes of planets based on historical optical observations of the Sun and planets. On the other hand, by taking into account the significantly different time intervals covered by the optical observations, a comparison of the results can provide insight into the degree of reliability that should be assigned both to new and to traditional methods.

In reviewing the classical problem of nonprecessional equinox motion, Newcomb's studies should primarily be mentioned (Newcomb 1895). He was the first to discuss this question and concluded from observations of the 18th and 19th centuries that the motion of the equinox is negligible. Later on, numerous analyses of observations from the first part of the 20th century showed a rather large negative correction to Newcomb's equinox, but they gave *no clear evidence* of its secular motion (see, e.g., Kahrstedt 1932; Morgan 1932, 1948, 1952; Fricke et al. 1963). So throughout two centuries, up to the 1970s, there was no strong observational evidence of the secular equinox drift based on optical observations of the Sun and the planets.

Ironically, dynamical determinations of the motion of the equinox have for a long time been in sharp discordance with the statistical study of stellar proper motions during the 20th century, indicating a significant nonprecessional term—for example, the study by Oort (1943) based on the FK3 motions yielded a correction $\Delta e + \Delta\lambda = 1''.19 \pm 0''.10$ per century, and that by Fricke (1967) based on the FK4 motions gave $\Delta e + \Delta\lambda = 1''.20 \pm 0''.11$ per century.

This discrepancy was eventually resolved by Fricke (1982), who determined the equinox motion by the dynamical method only after rejecting all 19th century equinox determinations. With the transition to the IAU 1976 system, the FK4 proper motions have been corrected by $1''.27$ per century, and it is now known from direct comparison of the FK5 with *Hipparcos* that the residual rotation of the ground-based motions is within $0''.1$ per century in absolute value.

However, in almost all cases optical observations reduced to the FK5 system and compared with the numerical ephemerides have provided results in *disagreement* with the expected small rotation of the FK5. The discussions by Yao & Smith (1988, 1991, 1993), Krasinsky et al. (1993), Standish & Williams (1990), Seidelmann et al. (1985, 1986), Seidelmann (1992), Kolesnik (1995, 1996), and Poppe et al. (1998) have shown that the residuals in the right ascension of the Sun exhibit a nearly $1''$ per century negative linear drift before 1960 and an equivalent positive drift after that date. No intelligible explanation of these trends in the optical observations has yet been given.

However, since it is now certain that the rotation of the stellar reference system is small, it has become obvious that, when compared with the integrated ephemerides, the optical observations of the Sun and planets show something other than merely a residual rotation of the stellar frame with respect to the dynamical frame. The origin of the large discrepancy should be looked for elsewhere.

The authors who discussed the previous optical observations of the solar system bodies analyzed data based on a limited time span and used a limited subset of the instrumental series. In view of the relatively low accuracy of optical observations, this caused widespread skepticism regarding the reported results. These results were generally disregarded by the astronomical community, since they were dramatically inconsistent with results provided by more precise methods of observation. To provide new insight into the problem of the large discrepancy of secular trends shown by optical observations, special attention should be paid to extending the time span of observations as far as possible beyond the 20th century. A maximum diversity of instrumental series should be used to reduce the systematic errors of individual series, and correct procedures for reducing the historical observations onto the actually adopted stellar reference frame should be developed.

Another traditional application of solar and planetary optical observations is the determination of the tidal acceleration of the Moon. This was first estimated by Clemence (1948), who used the results of Spencer Jones (1939) on the apparent accelerations of the longitudes of the Sun, Mercury, and Venus observed with respect to Universal Time. Clemence proposed the introduction of an empirical term into a purely gravitational lunar theory to account for the tidal acceleration and to adjust the origin of the longitudes and the timelike argument of planetary ephemerides to the ephemeris of the Moon. Morrison (1979) amended the Jones-Clemence empirical correction, and this was applied by Stephenson & Morrison (1984) to the $j = 2$ Improved Lunar Ephemeris (ILE; Eckert et al. 1954) to form the differences ET − UT between Ephemeris Time and Universal Time that are now accepted by all astronomical almanacs. Since Spencer Jones's study, a large number of optical observations of the Sun and planets have been accumulated and the nonrotating *Hipparcos* reference frame and more precise integrated ephemerides of the planets have become available. With these achievements it is reasonable to revise the Jones-Clemence result and compare it with estimates supplied by modern techniques.

Lastly, it is of interest to estimate the consistency of the numerical ephemerides, which are based on a limited recent time span of radar ranging, with the traditional optical observations extended backward in time.

2. OBSERVATIONS

To meet the requirements described above, we attempted to incorporate almost all daytime observations of the Sun, Mercury, and Venus accumulated during the historical period of classical astrometry. In our estimation, the observational data used in this study make up about 90% of the total angular position measurements of these objects made from the surface of Earth. The instrumental series used here are presented in Table 1. References to the original publications (about 200 in number), as well as a detailed description and analysis of the individual series, will be given elsewhere. The observations represent the apparent right ascensions and declinations extracted from the original publications. The total number of observations in both right ascension and declination is 244,960.

The observational material can be naturally divided into three periods: 1750–1830, 1830–1900, and 1900–2000.

1. In the first period, general principles for angular measurements with meridian instruments were established, but it was only after the construction of Bessel's universal theory of reductions and methods of determination of instrumental constants that it became possible to reduce these observations more or less correctly. As can be seen from the table, Greenwich, Radcliffe, and Paris observations from that period were used in the subsequent reductions by Bessel, Airy, Le Verrier, Auwers, and Knox-Shaw.

2. In the second period, the quality of instruments and investigative methods were progressively improved, mainly by Pulkovo, Greenwich, and Washington astronomers, and observations of the daytime objects reached nearly $1''$ accuracy.

3. At the beginning of the 20th century, the moving-wire micrometer was introduced into common practice, significantly reducing personal errors. New kinds of instrumental corrections were identified and carefully applied. Observations of planets were more accurately linked to a reference catalog. Polar motion was allowed for in the declinations of planets. This resulted in greater accuracy and improved consistency of different instrumental series.

The typical internal precision of instrumental series in the 18th century observations of the daytime objects (Bradley, Maskelyne, Hornsby) was in this study estimated at $2''$ in right ascension and $1\farcs5$ in declination. In the early 19th century, Bessel observed with $1\farcs3$ accuracy in both right ascension and declination, while the best instruments of the second half of the century (Pulkovo, USNO) reached some $0\farcs5$–$0\farcs8$. The $0\farcs5$ level has remained typical for 20th century observations. Below, it will be demonstrated that, combined with the large amount of data and the time span of several centuries, even such low-precision measurements can give estimates of the secular trends comparable in accuracy to those provided by modern methods of observations made over a limited time span.

3. TRANSFORMATION OF OBSERVATIONS ONTO THE ICRS FRAME

Optical positions of the Sun and planets are differentially observed with respect to stars in some reference catalog. The published results ($\alpha_{\rm obs}$, $\delta_{\rm obs}$) are apparent places given in the system of such a catalog. Transformation of these positions onto the ICRS is therefore equivalent to (1) transforming the respective reference catalog to the ICRS and (2) taking into account the differences between the modern and historical astronomical constants used by observers for the calculation of the apparent places of stars, since the apparent places of the comparison ephemerides are calculated with the modern precession, nutation, and aberration constants.

For observations preceding the IAU 1976 standards, the stars to which observations are referred usually have their mean positions ($\alpha_{\rm st}$, $\delta_{\rm st}$) given at the beginning of a certain Besselian year $T_{\rm B}$. If a historical fundamental catalog is used, its proper motions ($\mu_{\rm st}$, $\mu'_{\rm st}$) are to be taken into account when observations are referred to this catalog on the epoch of observation t.

Several procedures aimed at transforming FK4-based mean positions to FK5-based positions have been proposed (Standish 1982; Aoki et al. 1983; Lederle & Schwan 1984; Smith et al. 1989). Since the scope of the present study is not limited to FK4-based observations, and since the data should be transformed to the ICRS, a new transformation method needs to be developed that may be universally applied to the variety of astronomical systems used over the observational history of astronomy.

The approach proposed here is based on direct comparison of a list of reference stars with the ICRS-based catalog positions ($\alpha_{\rm H}$, $\delta_{\rm H}$), rotated from T_0 = J2000 to the epoch of a reference catalog $T_{\rm B}$ by use of a modern precession constant and *Hipparcos* proper motions ($\mu_{\rm H}$, $\mu'_{\rm H}$). Let vector $\boldsymbol{r}_{\rm H}$ be the ICRS-based position at T_0 and $\boldsymbol{m}_{\rm H}$ be the *Hipparcos*-based proper motion formed in the usual way from $\alpha_{\rm H}$, $\delta_{\rm H}$, $\mu_{\rm H}$, and $\mu'_{\rm H}$ (see, e.g., Aoki et al. 1983). Let $\mathbf{P}_{\rm S}$ be the modern (Simon et al. 1994) precession matrix; then the vector

$$\boldsymbol{r}_{\rm H}\{T_{\rm B}\} = \mathbf{P}_{\rm S}\{T_0 \to T_{\rm B}\}[\boldsymbol{r}_{\rm H} + \boldsymbol{m}_{\rm H}(T_{\rm B} - T_0)]$$

TABLE 1
INSTRUMENTAL SERIES OF OPTICAL OBSERVATIONS

Instrumental Series	Objects	Years
Greenwich: Bradley (Auwers)	Sun, Mercury, Venus	1750–1762
Greenwich (Le Verrier)	Sun (α)	1750–1850
Greenwich (Airy)	Mercury, Venus	1750–1830
Greenwich	Sun, Mercury, Venus	1830–1954
Radcliffe: Hornsby (Knox-Shaw et al.)	Sun, Mercury, Venus	1774–1798
Königsberg (Le Verrier)	Sun (α)	1814–1830
Königsberg (Bessel)	Sun	1814–1848
Königsberg	Mercury, Venus	1839–1847
Dorpat (Lyapunov)	Sun	1823–1838
Paris	Sun, Mercury, Venus	1800–1935
Cambridge	Sun, Mercury, Venus	1828–1869
Edinburgh	Sun, Mercury, Venus	1834–1845
Radcliffe	Sun, Mercury, Venus	1840–1899
Berlin	Sun, Mercury, Venus	1838–1842
Pulkovo transit instrument	Sun (α)	1842–1915
Pulkovo vertical circle	Sun (δ)	1842–1914
USNO 4 inch	Sun, Mercury, Venus	1861–1865
USNO 8.5 inch	Sun, Mercury, Venus	1866–1891
USNO 9 inch	Sun, Mercury, Venus	1894–1945
USNO 6 inch	Sun, Mercury, Venus	1899–1982
Cape	Sun, Mercury, Venus	1861–1959
Besançon	Sun, Mercury, Venus	1890–1895
Strasbourg	Sun, Mercury, Venus	1882–1893
Odessa transit instrument	Sun (α)	1899–1903
Odessa vertical circle	Sun (δ)	1901–1910
Toulouse	Mercury, Venus	1912–1924
Uccle	Sun, Venus	1928–1932
Ottawa	Sun, Mercury, Venus	1924–1935
Nikolaev transit instrument	Sun, Mercury, Venus (α)	1929–1989
Nikolaev vertical circle	Sun, Mercury, Venus (δ)	1929–1985
Herstmonceux	Sun, Mercury, Venus	1957–1982
Pulkovo transit instrument	Sun, Mercury, Venus (α)	1956–1976
Pulkovo vertical circle	Sun, Mercury, Venus (δ)	1956–1976
Tashkent	Sun, Mercury, Venus (α)	1960–1992
Kiev (Goloseevo)	Sun, Mercury, Venus (δ)	1971–1987
Moscow	Sun, Mercury, Venus	1961–1975
Belgrade	Sun, Mercury, Venus	1974–1993
Kislovodsk-Pulkovo transit instrument	Sun, Mercury, Venus	1992–1998
Kislovodsk-Pulkovo vertical circle	Sun, Mercury, Venus	1988–1998
CERGA (astrolabe)	Sun (α)	1976–1988
San Fernando (astrolabe)	Sun (α)	1991–1992
Santiago (astrolabe)	Sun (α)	1990–1999
Simeiz (astrolabe)	Sun (α)	1987–1991

is the mean position of the *Hipparcos* star at epoch $T_{\rm B}$, which, after decomposition, corresponds to the respective right ascension and declination $\alpha_{\rm H}(T_{\rm B})$ and $\delta_{\rm H}(T_{\rm B})$. Elliptic terms of aberration are subtracted beforehand from ($\alpha_{\rm st}$, $\delta_{\rm st}$) in the way proposed by Smith et al. (1989). Residual differences are then formed with respect to a standard-star list with N entries $i = 1, \ldots, N$:

$$\Delta\alpha_i = \alpha_{\rm H}(T_{\rm B}) - \alpha_{\rm st}, \quad \Delta\delta_i = \delta_{\rm H}(T_{\rm B}) - \delta_{\rm st}.$$

These are approximated by using the spline smoothing technique (Wahba 1990; Kolesnik 2004), providing systematic differences $\Delta\alpha(\alpha, \delta)$ and $\Delta\delta(\alpha, \delta)$ at any point of the sky. The resulting spline approximation simultaneously accounts for systematic distortions of the reference catalogs with respect to the *Hipparcos* system extrapolated to the epoch $T_{\rm B}$, as well as for the offsets between catalogs due to a variety of factors, such as the difference between historical and modern precession models, the inaccuracy of the zero point in right ascension or declination of a reference catalog, etc. Analogously, systematic differences $\Delta\mu(\alpha, \delta)$, $\Delta\mu'(\alpha, \delta)$ in proper motions are formed for all historical fundamental catalogs (FK5, FK4, FK3, N30, GC, PGC, N2, Le Verrier). The constant offset of the motions in right ascension accounts for the nonprecessional equinox drift.

The initial correction transforming the observed positions of the Sun and planets to the *Hipparcos* system is

$$\Delta\alpha_1 = \Delta\alpha(\alpha_{\rm obs}, \delta_{\rm obs}) + \Delta\mu(\alpha_{\rm obs}, \delta_{\rm obs})(t - T_{\rm B}),$$
$$\Delta\delta_1 = \Delta\delta(\alpha_{\rm obs}, \delta_{\rm obs}) + \Delta\mu'(\alpha_{\rm obs}, \delta_{\rm obs})(t - T_{\rm B}).$$

Here $\Delta\alpha_1$ and $\Delta\delta_1$ form the vector $\Delta\boldsymbol{r}_1$ in the usual way. If the observations are referred to any individual catalog without proper motions, the second terms are absent in the above expressions. As an example, we illustrate in Figure 1 plots of the spline approximations for the *Hipparcos* minus N2 (epoch

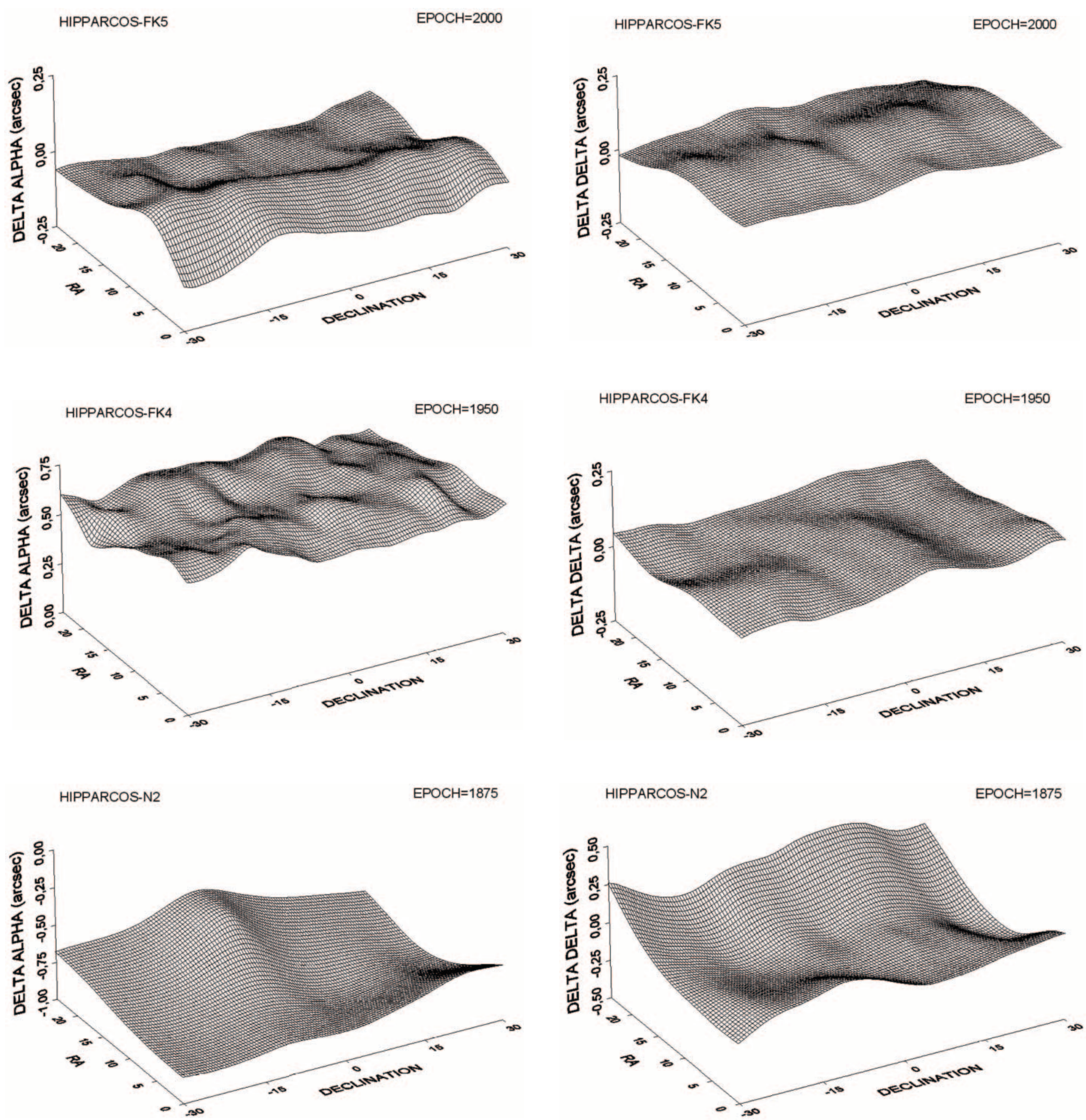

FIG. 1.—An example of the spline approximations for the systematic differences *Hipparcos* − N2 (epoch 1875), *Hipparcos* − FK4 (epoch 1950), and *Hipparcos* − FK5 (epoch 2000) applied for transformation of the respective historical observations of the Sun and planets to the ICRS.

FIG. 2.—The modified series EMT − UT = ET − UT + 1.821(−10″.26 − 24″.41T − 13″.00T^2) used in the comparison of the optical observations with the DE405 ephemeris.

1875), *Hipparcos* minus FK4 (epoch 1950), and *Hipparcos* minus FK5 (epoch 2000) position differences.

The next set of corrections is aimed at adjusting the modern and historical astronomical constants used by observers for the calculation of the apparent places of stars. The accumulated differences in precession are corrected for the interval between t and T_B. If $\boldsymbol{r}_{obs}$ is the vector formed from (α_{obs}, δ_{obs}), then

$$\Delta \boldsymbol{r}_2 = \mathbf{P}_S\{T_B \to t\}\boldsymbol{r}_{obs} - \mathbf{P}_{his}\{T_B \to t\}\boldsymbol{r}_{obs},$$

where $\mathbf{P}_{his}$ is the matrix formed from the precession angles adopted in a historical period.

The correction due to nutation is calculated as follows:

$$\Delta \boldsymbol{r}_3 = \mathbf{N}(t)\boldsymbol{r}_{obs} - \mathbf{N}_{his}(t)\boldsymbol{r}_{obs},$$

where $\mathbf{N}$ is the nutation matrix formed from the fundamental angles $\Delta\psi(t)$ and $\Delta\epsilon(t)$, calculated on the basis of a modern nutation series (Herring 1991) with 106 terms, and $\mathbf{N}_{his}$ is the respective matrix based on the historical nutation series.

The difference in aberration is corrected by simply calculating differences of the Bessel numbers,

$$\Delta C = -(20''.495 - k_{his})\cos L \cos\epsilon,$$

$$\Delta D = -(20''.495 - k_{his})\sin L$$

with k_{his} being the historical value of aberration, and applying them in the usual way at the positions of observed planets $\boldsymbol{r}_{obs}$, thus obtaining the correction $\Delta \boldsymbol{r}_4$.

The systematic differences varying with right ascension and declination, derived in the equatorial zone of the sky from $\Delta \boldsymbol{r}_1$ and $\Delta \boldsymbol{r}_2$, converted into equatorial coordinates, and interpolated onto observed positions $\boldsymbol{r}_{obs}$, are combined with the other three corrections: $\Delta \boldsymbol{r} = \Delta \boldsymbol{r}_1(\boldsymbol{r}_{obs}) + \Delta \boldsymbol{r}_2(\boldsymbol{r}_{obs}) + \Delta \boldsymbol{r}_3 + \Delta \boldsymbol{r}_4$. These have been applied to the optical observations of the Sun and the planets, for each series, taking into account the standard catalogs and astronomical constants to which they were referred.

Additional corrections, such as day-night differences, personal equations, and in some cases catalog offsets quoted in publications but for some reason not included in published positions, have also been applied. For Venus and Mercury, phase corrections were determined using the method presented in Kolesnik (1995) and subtracted from the residuals. For long instrumental series, these corrections were determined at short intervals usually associated with a certain catalog. Declination residuals on the interval 1848–1900 were also corrected for the effects of polar motion by use of the (x, y)-parameters derived by Rykhlova (1970).

As a reference catalog representing the ICRS frame we used the *Hipparcos* Catalogue. Systematic differences from the *Hipparcos* Catalogue giving the corrections $\Delta \boldsymbol{r}_1$ and $\Delta \boldsymbol{r}_2$ have been formed for about 100 standard-star lists and individual catalogs and eight fundamental catalogs associated with the respective series of observations. These were applied as indicated above.

4. COMPARISON PROCEDURE AND METHOD OF ANALYSIS

DE405 was used as the ephemeris for comparison. In order to be free from any assumptions about the tidal acceleration of the Moon in the comparison procedure (and hence the tidal spin-down of Earth), we used the modified ET − UT series of Stephenson & Morrison (1984) by eliminating the tidal term derived earlier by Morrison (1979):

$$\mathrm{EMT} - \mathrm{UT} = \mathrm{ET} - \mathrm{UT} + 1.821(-10''.26 - 24''.41T - 13''.00T^2).$$

The EMT − UT series formed in this way is shown in Figure 2. Using this modified series, only irregular fluctuations of Earth's rotation are taken into account, so that the final result in the quadratic secular trends should be interpreted as the *combination* of the tidal acceleration of Earth's rotation, errors in the ephemerides, and, possibly, dynamical modeling errors. Rectangular positions from DE405 were interpolated at the moments $\mathrm{JD_{EMT}} = \mathrm{JD_{UT}} + \Delta T/86,400$ with $\mathrm{JD_{UT}} = \mathrm{JD_0} + \mathrm{fd}$, where $\mathrm{JD_0}$ is the Julian Date at 0 hours UT and fd is the fractional part of the mean solar day calculated from the longitude of an instrument λ, observed right ascension α, and Greenwich Apparent Sidereal Time at 0 hours UT. A set of observed minus calculated ($O-C$) residuals formed the basic material of the analysis: $\Delta\alpha = \alpha_{\rm obs} - \alpha_{\rm calc}$, $\Delta\delta = \delta_{\rm obs} - \delta_{\rm calc}$. They are shown in Figure 3.

In a previous analysis of the problem (Kolesnik 2001a, 2001b), secular variations of the corrections to the orbital elements were determined separately from the right ascension and declination residuals. Such an approach was applied because analysis of the longitude corrections has shown significant differences in the results derived from right ascensions and declinations before 1900 due to systematic errors in the observations. However, after a careful analysis of the weight of the unknowns in the conditional equations, we report in this study the *combined* solution. It will be shown below that in such a solution, the systematic difference between right ascension and declination observations is completely absorbed by the equinox correction, while corrections to the longitudes of the Sun and planets are derived in the pure form. For the Sun the classical conditional equations are used:

$$\begin{aligned}\Delta\alpha &= \Delta L_0 \cos\epsilon \sec^2\delta + 2\Delta h \sin\alpha \sec\delta \\ &\quad - 2\Delta k \cos\alpha \sec\delta \cos\epsilon - \Delta\epsilon \cos\alpha \tan\delta - \Delta E, \\ \Delta\delta &= \Delta L_0 \sin\epsilon \cos\alpha + \Delta\epsilon \sin\alpha + 2\Delta h \sin\delta \cos\alpha \\ &\quad - 2\Delta k \cos^2\alpha \cos\delta \sin\epsilon + \Delta\delta_0. \end{aligned} \tag{1}$$

For Mercury and Venus, these are as follows:

$$\begin{aligned}\Delta a \cos\delta &= \sum_{i=1}^{4} \frac{\partial a}{\partial E_i^0} \Delta E_i^0 + \sum_{i=1}^{5} \frac{\partial a}{\partial E_i} \Delta E_i \\ &\quad + \Delta\Phi_a \sin\theta \cos i - \Delta E \cos\delta, \\ \Delta\delta &= \sum_{i=1}^{4} \frac{\partial\delta}{\partial E_i^0} \Delta E_i^0 + \sum_{i=1}^{5} \frac{\partial\delta}{\partial E_i} \Delta E_i \\ &\quad + \Delta\Phi_\delta \sin\theta \cos i - \Delta\delta_0, \end{aligned} \tag{2}$$

where the ΔE_i^0 are corrections to the mean elements of Earth's orbit, the ΔE_i are corrections to the elements of the respective planet including the longitude corrections, Φ_α and Φ_δ are phase corrections, $\Delta\delta_0$ is a constant offset in declination, and ΔE is the equinox correction. Numerical expressions for the coefficients are given in Kolesnik (1995). Hereafter ΔL_0 and ΔL will denote corrections to the mean longitudes of Earth and the respective planet. In view of the large systematic discrepancy in declination offsets of the numerous instrumental series throughout two and a half centuries, the equator offset was not estimated. The constant errors in the equator were determined for each series and have been subtracted. Phase corrections were determined for each series and subtracted as well.

Corrections to the orbital elements were derived by solving the conditional equations (eqs. [1]–[2]) in relatively short time bins. This approach was applied, on the one hand, to demonstrate graphically the secular trends in the longitude corrections over the 250 year time span that are not visible from residuals in right ascension and declination themselves. On the other hand, such an approach has an advantage over a direct least-squares fit to the residuals, since it provides one the opportunity to take into account the changing accuracy of the observations throughout the historical period, by assigning respective weights to the solutions in bins. The time intervals of the bins were chosen to be 3 yr for the Sun, 3.63 yr for Mercury, and 3.075 yr for Venus, as multiples of the respective orbital periods. The number of residuals in each bin for the three objects are shown in Figure 4. The unit weight errors of the combined solution in bins are shown in Figure 5. With such numbers, the formal errors of the longitude corrections in the bins are usually below 0.″1 after 1850 and between 0.″1 and 0.″5 before that date. The dispersions of individual solutions generally confirm these internal error estimates.

The stepwise solution scheme implies that different instrumental series are randomly combined in each bin. The individual contribution of each series to the joint solution was iteratively estimated, and weights were assigned and applied to instrumental series on the basis of the dispersion of residuals with respect to the common solution.

5. SECULAR TRENDS IN CORRECTIONS TO THE MEAN LONGITUDES OF THE SUN, MERCURY, AND VENUS, AND THE EQUINOX CORRECTION

The individual-bin solutions in the 1750–2000 time interval tracing the secular variations of the longitudes of Earth, ΔL_0, and the planets, ΔL are presented in the subsequent plots, accompanied by second-order polynomial approximations. Figure 6 shows the variation of ΔL_0 as derived from the combined solutions on the basis of observations of the Sun, Mercury, and Venus. Figure 7 shows the analogous trends and their approximations in the longitudes of Mercury and Venus. The approximations represent the weighted solutions with weights inversely proportional to the squares of the respective formal errors in longitude in bins, shown as error bars in Figures 6 and 7. Table 2 provides numerical estimates of the trends in longitude with the constant A_0, linear A_1, and quadratic A_2 terms of the second-order polynomials. Equinox corrections and the respective linear approximations derived from the Sun, Mercury, and Venus are presented in Figure 8.

The analysis of the plots and the numerical estimates in Table 2 can be summarized as follows: individual solutions for ΔL_0 derived from *all three objects* are very consistent. When comparing the quadratic terms A_2 for the Sun, Mercury, and Venus, it can be seen that they are nearly proportional to the apparent diurnal mean motions of the respective planets.

Traditionally, the secular bias in residuals of differential optical observations of the Sun and planets results from the following factors: (1) constant error in the proper motions of a reference catalog (residual rotation with respect to the dynamical reference frame); (2) the effect of the tidal acceleration of Earth; (3) secular errors of the longitudes in a comparison ephemeris; and (4) systematic observation errors. An alternative explanation is a planetary secular acceleration of cosmological origin (Masreliez 1999).

Preliminary interpretations of the secular variation of the equinox corrections resulting from separate solutions can be found in Kolesnik (2001a). The equinox motion was there determined as a simple difference of the linear terms of the respective fits. In the present study we have applied the combined solution and approximated the equinox motion in the interval 1750–2000.

The good consistency of the ΔL_0 corrections in the 20th century compared with the results from the 18th and 19th centuries, shown in the aforementioned paper, provides evidence of systematic observational errors in these earlier epochs. We conclude that estimates of the motion of the equinox are meaningless over the whole interval of historical observations.

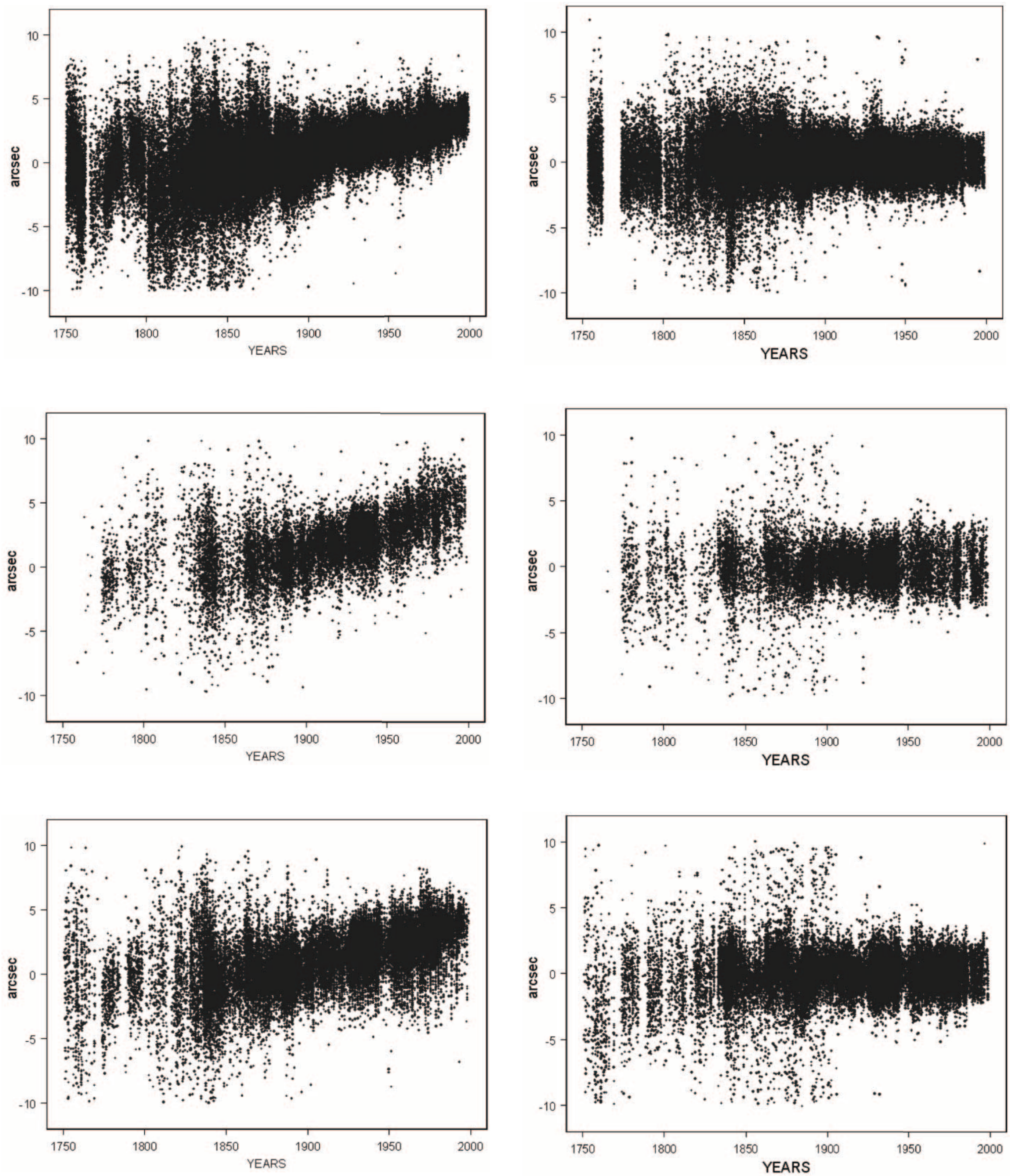

FIG. 3.—Residuals of the right ascensions (*left*) and declinations (*right*) of the Sun (*top*), Mercury (*middle*), and Venus (*bottom*) transformed to the ICRS as compared with DE405.

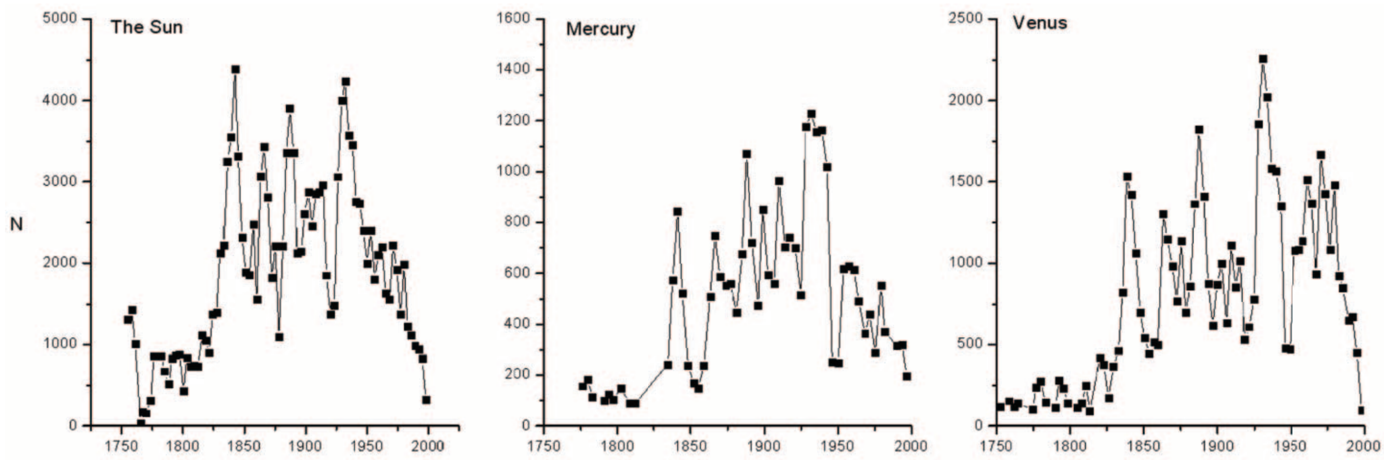

FIG. 4.—Number of observations in the 3 yr bin solution for the Sun, Mercury, and Venus.

This can clearly be seen in the plot of the equinox corrections in Figure 8. The linear fits shown in this plot suggest systematic errors in right ascension in the 19th century, as shown by observations of the Sun and both planets, and certainly have nothing in common with the rotation of the *Hipparcos* system.

It can also be seen from the plots that the constant ΔE term in the 19th century is close to Newcomb's equinox, while the linear term of the equinox determined from the Sun is close to Fricke's equinox motion, both having opposite signs. These results simply explain the origin of the large offset of the fundamental systems before the FK3, as well as their residual rotation affecting the proper motions up to FK5. Actually, these are caused by the systematic difference between observations of the Sun in right ascension and declination to which individual catalogs of the 18th and 19th centuries were tied. This difference progressively decreased with time throughout 150 years, causing a corresponding diminution of errors in proper motions derived from comparison of star positions in individual catalogs.

It is worth mentioning that the equinoxes derived from Mercury and Venus in the 19th century are significantly different. For example, Mercury does not show any equinox motion at all, and this motion estimated from Venus is only half as large as that from the Sun. Observations of Mercury and Venus were almost ignored at that epoch in the determination of the equinox.

Traditionally, this systematic difference and its evolution over time are ascribed to the different methods used for right ascension observations, which could cause personal errors of a group of observers when recording transits of the Sun and reference stars. Indeed, the transition from the "eye and ear" method to "chronographic" and then to "contact micrometer" for some instrumental series produces discontinuity in the $O-C$ residuals. In the course of this study, such discontinuities were investigated and estimated for observations of the Sun and both planets. But it has been found that the shift of most of them is much smaller than the actually observed difference in longitude residuals, which reaches some $0''.5$–$1''$. In this context it is worth mentioning the experiments made throughout 30 years at the US Naval Observatory with a personal-equation machine at a 9 inch transit circle (Eichelberger & Morgan 1920; Morgan 1933; Morgan & Scott 1948) and a 6 inch transit circle (Hammond & Watts 1927; Watts & Adams 1949). Throughout these years, experiments were made by many observers, measuring their absolute personal equation with respect to artificial stars. The results are striking. From the tables, which can be found in the aforementioned publications, it is clear that for all observers the transition from the "eye and ear" to the chronograph method and then to the moving-wire method caused time differences of 0.1–0.2 s in the observations of day stars and the Sun. The difference in the personal equation

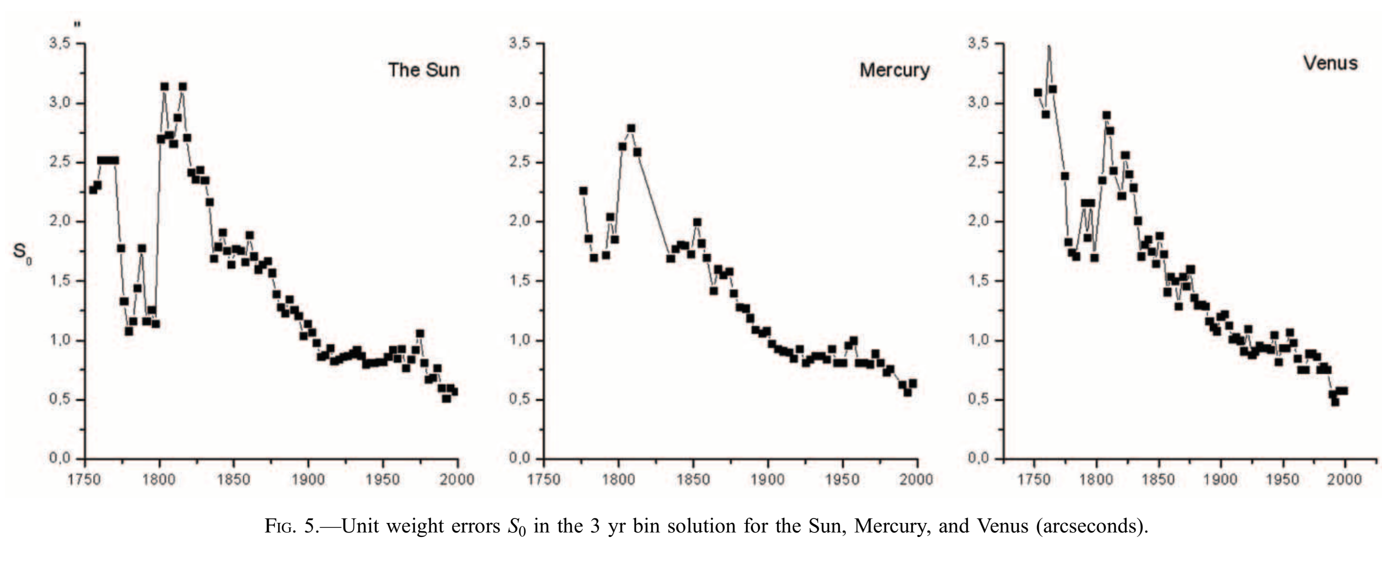

FIG. 5.—Unit weight errors S_0 in the 3 yr bin solution for the Sun, Mercury, and Venus (arcseconds).

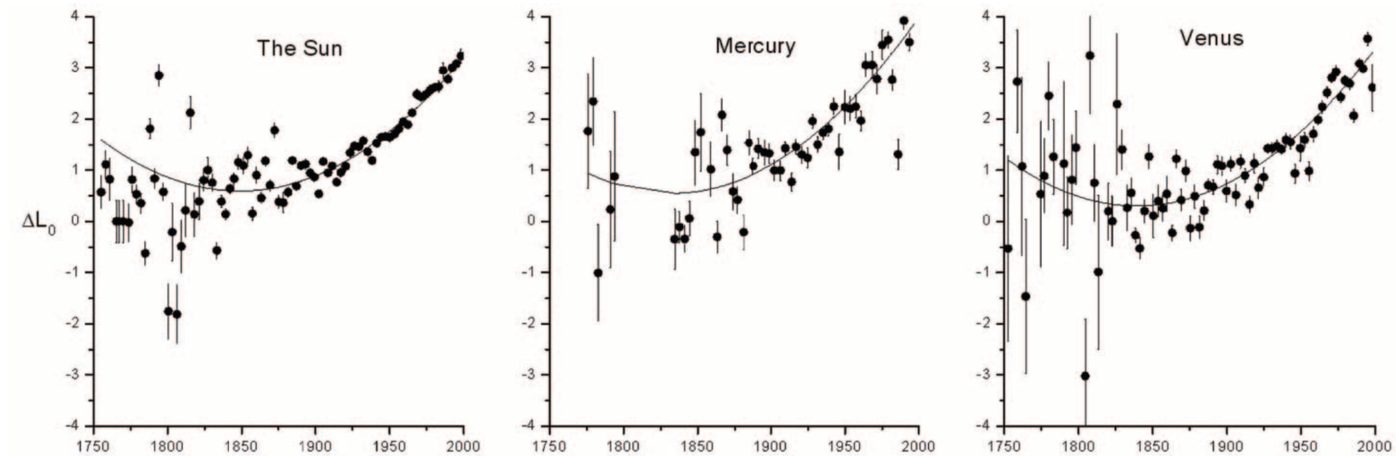

FIG. 6.—Corrections to the longitude of Earth, ΔL_0, derived from the combined solution (R.A. + decl.) of observations of the Sun, Mercury, and Venus (in arcseconds). The solid line represents the weighted second-order approximation.

for a given individual between day stars and planets is considerably smaller. All these results are consistent with the plots of the historical equinox corrections presented in this study.

Indeed, while using the same method the peculiar features of a group of observers, who worked with the same instrument, are averaged, but the feature of the method itself (''eye and ear,'' ''fixed threads,'' ''moving threads'') can, in principle, influence all observers the same way and in the same direction. For example, it is quite possible that when using the ''tapping method,'' the human perception might be constituted such that there can be a systematic delay in time recording compared with the method when an observer uses a ''moving thread.'' Moreover, this delay can be similar for every human, since it depends on the constitution of our nervous system, so that personal errors would not be averaged out within the group of observers. As a result, observations of the Sun made with all instruments using the same method will be biased approximately by the same systematic error and in the same direction. That is probably what we see in the right ascension plot of the longitude corrections presented in Kolesnik (2001a), and this inevitably affects the equinox correction.

A preliminary evaluation of the equinox motion from observations of the 20th century was made by Kolesnik (2001a). The residual orientation and rotation of the *Hipparcos* frame can be estimated from corrections to the inclination of the ecliptic $\Delta\epsilon$, corrections to the mean longitude of Earth ΔL_0, and equinox corrections ΔE, and their derivatives in the following way:

$$\epsilon_x = \Delta\epsilon, \quad \epsilon_y = -\Delta L_0 \sin\epsilon, \quad \epsilon_z = \Delta L_0 \cos\epsilon - \Delta E,$$
$$\omega_x = \Delta\dot{\epsilon}, \quad \omega_y = -\Delta\dot{L}_0 \sin\epsilon, \quad \omega_z = \Delta\dot{L}_0 \cos\epsilon - \Delta\dot{E}.$$

Estimates of these angles from 20th century optical observations of the Sun, Mercury, and Venus are given in Table 3.

As for corrections to the longitudes of the planets, the data presented in Table 2 can be interpreted as a combination of the nongravitational term in the lunar theory, which is intentionally omitted here as indicated at the beginning of § 4, and a correction to the theoretical mean longitudes of the DE405 ephemeris. If the latter is regarded as error-free, the respective constant term transformed by the ratio of the mean motions has the meaning of an offset in the origin of longitudes in the ILE $j = 2$ theory with respect to the ICRS at a certain epoch. The linear term must be interpreted as a combination of corrections to the mean motion of the Moon, the motions of the respective planets, and the residual rotation of its origin with respect to the ICRS. The quadratic term represents the revised tidal semiacceleration of the Moon, but it may also be due to unmodeled planetary acceleration.

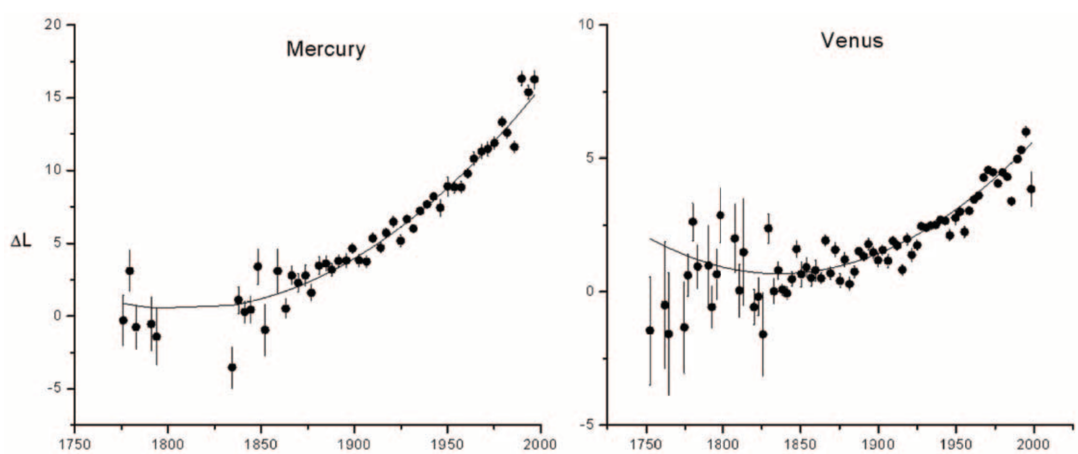

FIG. 7.—Same as Fig. 6, but for the longitudes of Mercury and Venus.

TABLE 2

TRENDS IN LONGITUDE

Object	A_0 (arcsec)	A_1 (arcsec)	A_2 (arcsec)
Sun (ΔL_0)	0.89 ± 0.03	1.16 ± 0.08	1.14 ± 0.11
Mercury (ΔL_0)	1.11 ± 0.06	1.66 ± 0.26	1.23 ± 0.31
Venus (ΔL_0)	0.74 ± 0.07	1.45 ± 0.11	1.19 ± 0.18
Mercury (ΔL)	3.96 ± 0.14	7.61 ± 0.34	4.14 ± 0.48
Venus (ΔL)	1.44 ± 0.06	2.38 ± 0.19	1.86 ± 0.23

NOTE.—Listed are the coefficients of the second-order approximation of the trends in the secular variations of the longitudes of Earth, ΔL_0, derived from observations of the Sun, Mercury, and Venus, and the longitudes of Mercury and Venus, ΔL. The mean epoch of the solutions is 1900.

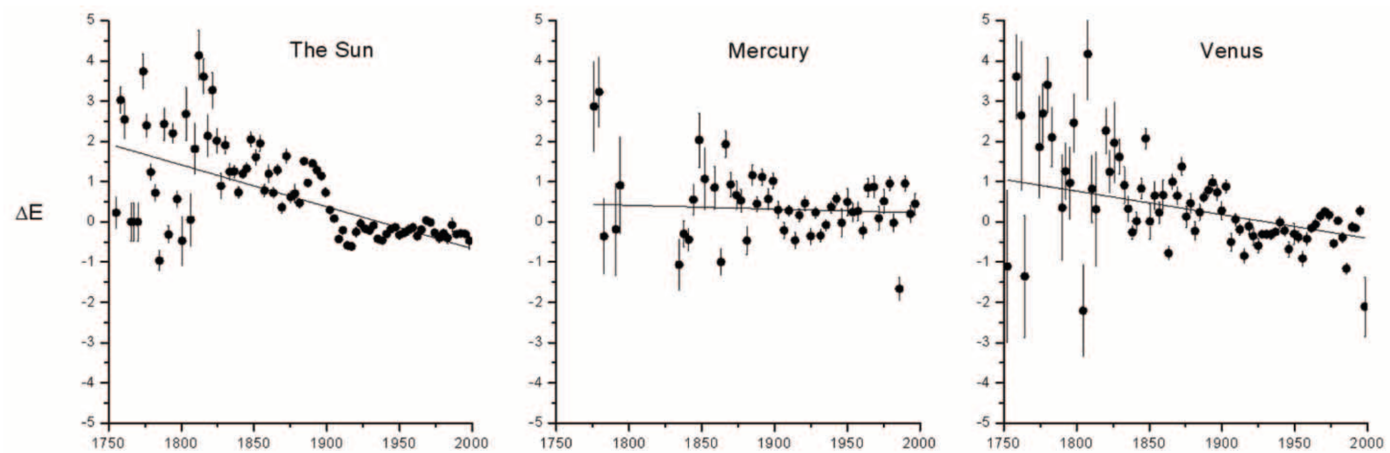

FIG. 8.—Equinox corrections derived from observations of the Sun, Mercury, and Venus (in arcseconds). The solid line is the linear approximation over the time span 1750–2000.

TABLE 3

ORIENTATION AND ROTATION ANGLES OF THE *Hipparcos* CATALOGUE WITH RESPECT TO DE405 AS DERIVED FROM OPTICAL OBSERVATIONS OF THE SUN, MERCURY, AND VENUS IN THE 20TH CENTURY

Object	Orientation (mas)			Rotation (mas yr^{-1})		
	ϵ_x	ϵ_y	ϵ_z	ω_x	ω_y	ω_z
Sun	67.4 ± 19.8	75.7 ± 13.5	−71.0 ± 47.6	3.90 ± 0.26	0.65 ± 0.26	−3.71 ± 0.94
Mercury	87.2 ± 10.4	77.4 ± 10.8	180.1 ± 46.1	1.85 ± 0.20	0.83 ± 0.20	0.66 ± 0.83
Venus	37.5 ± 10.4	89.0 ± 12.4	23.9 ± 41.8	−0.17 ± 0.21	1.14 ± 0.27	−1.43 ± 0.88

The longitude corrections, converted to the basis of the motion of the Moon by the respective ratios of the mean motions, are significantly different compared with the respective estimates made by one of the authors (Kolesnik 2001b) from right ascensions only. Evidently, our previous estimates are affected by the systematic errors in right ascension as discussed above. It should also be mentioned that the weighted mean of the tidal acceleration of the Moon, estimated in this study in absolute value, is about $4''$ century^{-2} larger than that determined from LLR measurements by Chapront et al. (2002) and Mercury transits by Morrison (1979).

On the other hand, if the magnitude of the tidal acceleration is under question, the excessive quadratic trends can possibly be interpreted as the result of applying an inaccurate model for the spacetime metrics adopted in the equations of motion, which also might influence the ephemeris creation process (see the respective theories of Dirac 1973; Canuto et al. 1977; Masreliez 1999).

The quadratic trends presented in Table 2 do not disagree significantly with corresponding trends based on a limited sample using 30 years of Atomic Time (Kolesnik, private communication reported in Masreliez 1999). This is surprising, since traditionally Universal Time decelerates significantly relative to Atomic Time, which should be reflected in the estimates of the secular accelerations. The interval with Atomic Time is too short to confirm this discrepancy; however, if it is real, planetary secular accelerations would be explained by the model proposed in Masreliez (1999).

6. CONCLUSION

In the present study, the unprecedented number of 245,000 optical observations of the Sun, Mercury, and Venus covering an interval of 250 years have been used to investigate the secular variation of the longitudes of Earth, Mercury, and Venus. The observations are reduced to the *Hipparcos*-based reference frame using a uniform reduction scheme for all objects and compared with the ICRS-based numerical ephemeris DE405. After elimination of the tidal term, numerical estimates of the discrepancy between observations and ephemerides, which are proportional to the mean motions of the Sun and planets, are presented. We conclude that the apparent equinox motion resulting from the 250 year interval of optical observations cannot be interpreted as an actual rotation of the *Hipparcos* system. More likely, it is caused by systematic errors in the right ascensions of the 19th century observations. In order to properly investigate the residual rotation of a reference catalog, only 20th century observations should be used. The tidal acceleration of the Moon estimated in this study is, in absolute value, about $4''$ century^{-2} larger than that determined from LLR measurements.

REFERENCES

Aoki, S., Sōma, M., Kinoshita, H., & Inoue, K. 1983, A&A, 128, 263
Canuto, V., Adams, P. J., Hseih, S.-H., & Tsiang, F. 1977, Phys. Rev. D, 16, 1643
Chapront, J., Chapront-Touzé, M., & Francou, G. 2002, A&A, 387, 700
Clemence, G. M. 1948, AJ, 53, 169
Dirac, P. A. M. 1973, Proc. R. Soc. London A, 333, 403
Eckert, W. J., Jones, R., & Clark, H. K. 1954, Improved Lunar Ephemeris 1952–1959 (Washington: GPO)
Eichelberger, W. S., & Morgan, H. R. 1920, Results of Obervations with the Nine-Inch Transit Circle, 1903–1911 (Publ. US Nav. Obs., 2nd Ser., 9, Part 1) (Washington: GPO)
Fricke, W. 1967, AJ, 72, 1368
———. 1982, A&A, 107, L13
Fricke, W., et al. 1963, Veröff. Astron. Rechen-Inst. Heidelberg, No. 10
Hammond, J. C., & Watts, C. B. 1927, Results of Observations with the Six-Inch Transit Circle, 1909–1918 (Publ. US Nav. Obs., 2nd Ser., 11, Part 1) (Washington: GPO)
Herring, T. A. 1991, in IAU Colloq. 127, Reference Systems, ed. J. A. Hughes, C. A. Smith, & G. H. Kaplan (Washington: US Nav. Obs.), 157
Kahrstedt, A. 1932, Astron. Nachr., 244, 33
Kolesnik, Yu. B. 1995, A&A, 294, 874
———. 1996, in IAU Symp. 172, Dynamics, Ephemerides and Astrometry of the Solar System, ed. S. Ferraz-Mello, B. Morando, & J.-E. Arlot (Dordrecht: Kluwer), 477
———. 2001a, in Systèmes de référence spatio-temporels: Journées 2000, ed. N. Capitaine (Paris: Obs. Paris), 119
———. 2001b, in Systèmes de référence spatio-temporels: Journées 2000, ed. N. Capitaine (Paris: Obs. Paris), 231
———. 2004, in Systèmes de référence spatio-temporels: Journées 2003, ed. N. Capitaine (Paris: Obs. Paris), in press
Krasinsky, G. A., Pitjeva, E. V., Sveshnikov, M. L., & Chunayeva, L. I. 1993, Celest. Mech. Dyn. Astron., 55, 1
Lederle, T., & Schwan, H. 1984, A&A, 134, 1
Masreliez, C. J. 1999, Ap&SS, 266, 399
Morgan, H. R. 1932, AJ, 41, 177
———. 1933, Results of Observations with the Nine-Inch Transit Circle, 1913–1926 (Publ. US Nav. Obs., 2nd Ser., 13, Part 1) (Washington: GPO)
———. 1948, AJ, 54, 1
———. 1952, AJ, 57, 232
Morgan, H. R., & Scott, F. P. 1948, Results of Observations Made with the Nine-Inch Transit Circle, 1935–1945 (Publ. US Nav. Obs., 2nd Ser., 15, Part 5) (Washington: GPO)
Morrison, L. V. 1979, MNRAS, 187, 41
Newcomb, S. 1895, The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy (Washington: GPO)
Oort, J. H. 1943, Bull. Astron. Inst. Netherlands, 9, 423
Poppe, P. C. R., Leister, N. V., Laclare, F., & Delmas, C. 1998, AJ, 116, 2574
Rykhlova, L. V. 1970, Soobsch. Gos. Astron. Inst. Shternberga, No. 163, 3
Seidelmann, P. K. 1992, in IAU Symp. 152, Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, ed. S. Ferraz-Mello (Dordrecht: Kluwer), 49
Seidelmann, P. K., Santoro, E. J., & Pulkkinen, K. F. 1985, in Dynamical Astronomy, ed. V. G. Szebehely & B. Balazs (Austin: Univ. Texas Press), 55
———. 1986, in IAU Symp. 114, Relativity in Celestial Mechanics and Astrometry, ed. J. Kovalevsky & V. A. Brumberg (Dordrecht: Kluwer), 99
Simon, J. L., Bretagnon, P., Chapront, J., Chapront-Touzé, M., Francou, G., & Laskar, J. 1994, A&A, 282, 663
Smith, C. A., Kaplan, G. H., Hughes, J. A., Seidelmann, P. K., Yallop, B. D., & Hohenkerk, C. Y. 1989, AJ, 97, 265
Spencer Jones, H. 1939, MNRAS, 99, 541
Standish, E. M., Jr. 1982, A&A, 115, 20
Standish, E. M., & Williams, J. G. 1990, in IAU Symp. 141, Inertial Coordinate System on the Sky, ed. J. H. Lieske & V. K. Abalakin (Dordrecht: Kluwer), 173
Stephenson, F. R., & Morrison, L. V. 1984, Phil. Trans. R. Soc. London A, 313, 47
Wahba, G. 1990, Spline Models for Observational Data (Philadelphia: Soc. Ind. Appl. Comput.)
Watts, C. B., & Adams, A. N. 1949, Results of Observations Made with the Six-Inch Transit Circle, 1925–1941 (Publ. US Nav. Obs., 2nd Ser., 16, Part 1) (Washington: GPO)
Yao, Z.-G., & Smith, C. 1988, in IAU Symp. 133, Mapping the Sky, ed. S. Débarbat, J. A. Eddy, H. K. Eichhorn, & A. R. Upgren (Dordrecht: Kluwer), 501
———. 1991, Ap&SS, 177, 181
———. 1993, in IAU Symp. 156, Developments in Astrometry and Their Impact on Astrophysics and Geodynamics, ed. I. I. Mueller & B. Kolaczek (Dordrecht: Kluwer), 403