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U CEPHEI: AN ANOMALOUS SPECTROGRAPHIC RESULT

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ABSTRACT

Thirty-four *spectrograms* of the *bright component* of the eclipsing variable U Cephei, in spite of broad, shallow lines and the resulting large probable error of a single observation (13 km/sec.), yielded, by reason of a large range in velocity (220 km/sec.), a satisfactorily defined velocity-curve, the elements from which (Table VI) indicate an orbit of *abnormally high eccentricity* (0.474) whose major axis is inclined approximately 65° to the line of sight. Earlier *photometric data*, largely influenced by a shallow secondary minimum near mid-phase, have indicated a circular orbit, but constancy of duration of eclipse and the nature of the variation of the period seem to preclude a reconciliation of the two sets of data by the assumption of a rotation of the line of apsides which should bring it into coincidence with the line of sight at the photometric epoch. The *period* is shown to increase steadily, with an oscillation of a few seconds' amplitude and an irregular period of about a dozen years. The usual *methods for photometric orbits* are *extended* to include cases where spectrograms have indicated orbits of high eccentricity.

Since its discovery by W. Ceraski in 1880, U Cephei has been one of the more thoroughly observed, photometrically, of the eclipsing variables, a long series of measures by various observers culminating in the extensive work of R. S. Dugan,[1] which yielded an orbit of high internal consistency. It therefore seems especially desirable to supplement these data with a spectroscopic orbit, and I was able to take spectrograms for this purpose at the Lick Observatory in 1923–1925, for which opportunity I am indebted to the kindness of Dr. Aitken. The only spectroscopic data available to me regarding this star, besides the H.D. classification[2] of primary and secondary

[1] *Princeton Contributions*, No. 5. Reference is made to this memoir for a very complete summary of earlier observations.

[2] See also A. J. Cannon, *Popular Astronomy*, **25**, 314, 1917.

components (Ao, Ko) and an earlier description of the spectrum at various phases by S. Blajko,[1] is V. M. Slipher's report[2] of a range of 90 km/sec. between two plates taken in 1907. Some other observers, however, have photographed the spectrum, and it is perhaps due to the discouraging character of the lines that no spectrographic orbit has heretofore been computed; but a large range in velocity fortunately renders the poor definition of the lines rather innocuous.

THE SPECTROGRAPHIC DATA

In view of the unexpected orbital eccentricity disclosed by the spectrograms, it appears well to describe the observations and reductions in rather more detail than would be required ordinarily. The plates were made with the 36-inch refractor, with use of a single-prism spectrograph with a 16-inch camera combination, giving a dispersion of 12.8 mm between $H\beta$ and $H\delta$. Exposures were generally 30–60 minutes in length, though a few were shorter. Only wide and diffuse hydrogen lines appear, three of which were ordinarily usable for the determination of velocity. Owing to the faintness of primary minimum, observations were confined to the phases of maximum light (6.8 mag.), and showed no trace of secondary spectrum, just as would be expected from the considerable range in brightness (2.3 mag.). From a plate taken with an exposure of 150 minutes during primary minimum, A. H. Joy very kindly provided a velocity of the faint component, based on seven lines. In conformity with the photometric data, a nearly circular orbit was expected, and it was intended to use only eighteen or twenty spectrograms, but when a preliminary plot showed evidence of considerable eccentricity, several more plates were taken at critical phases, making a total of thirty-four usable plates, including the one from Mount Wilson.

All of the spectrograms except Joy's were measured on a Gaertner comparator by Mrs. Carpenter (to whom I am also indebted for the greater part of the computing as far as orbit III), and many of the plates were measured by myself as well, the measurer being in no case aware of the phase of the plate. Table I presents the observational data. Since $H\beta$ and $H\gamma$ were always measured and $H\epsilon$ could

[1] *Annales de l'Observatoire de Moscou* (2d ser.), **5**, No. 9.

[2] *Astrophysical Journal*, **25**, 284, 1907.

be used only on exposures strong enough to show $H\delta$ as well, column 7 indicates the lines used for the velocities for each plate except Joy's, whose line identifications were not revealed. The weights

TABLE I

SPECTROGRAPHIC OBSERVATIONS OF U CEPHEI

Plate No.	Normal No.	Date	G.M.T.	J.D. 2420000+	Phase from Obs. Prim. Minimum*	No. Lines	Wt.	Veloc. Km/Sec.	O−C
12794....	8	1923 Jul. 18	23^h35^m	3619.983	2^d034	4	2	+126	− 6
Y 11942†...	12	21	22 10	3622.924	2.480	7	3	− 4	+12
12807....	7	23	21 6	3624.880	1.949	3	1	+134	+38
12808....	7	23	22 2	3624.918	1.989	3	1	+106	− 8
12809....	8	23	22 47	3624.949	2.019	3	1	+126	0
12810....	8	23	23 14	3624.968	2.039	3	2	+126	− 9
12843....	3	Aug. 6	21 39	3638.902	1.012	3	2	− 74	−20
12844....	3	6	22 38	3638.943	1.052	3	2	− 63	−11
12845....	3	6	23 51	3638.994	1.102	3	2	− 53	− 4
13102....	1	1924 Jan. 7	17 34	3792.731	0.282	3	1	− 65	− 3
13446‡...	5	Jun. 1	17 46	3938.740	1.703	3	2	+ 18	− 7
13447‡...	5	1	18 29	3938.770	1.733	3	1	+ 58	+26
13542....	4	Jul. 28	17 20	3995.720	1.346	3	2	− 27	+ 2
13543....	4	28	18 59	3995.791	1.416	4	2	− 31	−10
13549....	3	30	19 13	3997.801	0.935	3	2	− 63	− 5
13566....	9	Aug. 5	22 57	4003.956	2.102	3	2	+152	+ 4
13584....	10	21	0 53	4019.036	2.229	3	2	+138	+11
13715....	3	Oct. 30	22 32	4089.939	0.835	3	2	− 54	+ 8
13716....	3	30	23 47	4089.989	.885	3	2	− 50	+10
13756....	2	Nov. 19	.16 19	4109.680	.633	2	1	− 76	− 8
13757....	2	19	17 56	4109.747	0.700	2	1	− 64	+ 2
13832....	10	1925 Jan. 2	15 43	4153.655	2.224	2	1	+108	−18
13833....	11	2	17 20	4153.722	2.293	4	2	+ 57	−21
13846....	1	5	15 58	4156.665	0.247	3	2	− 46	+15
13847....	1	5	18 25	4156.767	0.356	3	2	− 85	−19
13899....	6	Feb. 18	16 5	4200.670	1.877	3	1	+ 93	+31
13901....	7	18	18 14	4200.760	1.967	3	1	+ 91	−12
13939....	1	Mar. 13	22 55	4223.955	0.229	3	1	− 79	−21
14007....	4	May 23	21 32	4294.897	1.375	2	½	+ 19	+45
14008V..	4	23	22 52	4294.952	1.430	2	½	− 26	− 6
14008W..	4	24	0 20	4295.014	1.492	2	½	+ 22	+34
14022....	5	Jun. 5	17 55	4307.746	1.770	4	3	+ 37	− 4
14023....	6	5	20 56	4307.872	1.896	4	3	+ 54	−24
14024....	7	5	22 30	4307.937	1.961	4	3	+106	+ 4

* Based upon earlier observations of Campbell (*H.B.*, No. 762).

† Mount Wilson observation of secondary component.

‡ Spectrograph provided with dense prism, dispersion = 18.1 mm between $H\beta$ and $H\gamma$.

listed in column 8 were assigned at the time of measurement, and are based upon the appearance of the plate and the consistency of its results. Table II, which requires no explanation, shows the combination of the observations into normal places.

The period of U Cephei, which will be discussed in more detail on a later page, has long been known to be variable. In reducing the spectrographic observations the latest available light-elements given

TABLE II

SPECTROGRAPHIC NORMAL PLACES

No.	Wt.	Phase from Obs. Prim. Minimum	Velocity Km/Sec.	O−C
1.........	0.50	$0^d.283$	− 67.8	− 5.3
2.........	0.17	.667	− 70.1	− 2.7
3.........	1.00	0.971	− 59.5	− 2.7
4.........	0.42	1.395	− 19.7	+ 4.2
5.........	.50	1.741	+ 34.2	+ 1.1
6.........	.33	1.892	+ 63.5	−13.0
7.........	.50	1.968	+107.8	+ 3.5
8.........	.42	2.034	+125.7	− 4.0
9.........	.17	2.100	+152.0	+ 3.0
10.........	.25	2.225	+128.0	+ 6.8
11.........	.17	2.291	+ 57.0	−20.9
12.........	0.25	2.479	− 4.1	+12.1

by Leon Campbell[1] as best satisfying observations from 1905 to 1922 were used:

$$\text{J.D. } 2423054.550 + 2^d.492901E\,,$$

but his later observations[2] showed that during the interval of the spectrographic observations it would be more satisfactory to use

$$\text{J.D. } 2424804.604 + 2^d.4929507E\,.$$

The effect of this change of the period upon the reduction of the spectrograms is inconsequential, although there is a slight progressive change in phase, which is shown in Table III. While the phases used originally are kept in Tables I and II, in the further discussion the effect of epoch has been practically removed by the application of a mean correction of −0.02 days to the phases from observed primary minimum.

A plot of the individual observations, reduced by the method of Lehmann-Filhés, yielded preliminary elements which, with the exception of the period, were improved by a least-squares solution,

[1] *Harvard Bulletin*, No. 762, 1922. [2] *Ibid.*, No. 842, 1927.

with employment of Schlesinger's usual formulae.[1] The corrections are given in the second column of Table V. Since some of the correc-

TABLE III

True Phases of Spectrograms from Primary Minimum *minus* Phases Used in Computing the Spectrographic Orbit

J.D.	Δ Phase
2423560	$-0^d.010$
2423810	.015
2423060	.020
2423310	−0.025

TABLE IV

Preliminary Elements

$P = 2^d.492901$ (adopted for the solution)
$\gamma = -5.0$ km/sec.
$K = 115.9$ km/sec.
$e = 0.451$
$\omega = 20^\circ.3$
$T =$ J.D. 2423966.682

tions seemed rather large, a second adjustment was carried out, the results of which are shown in columns 3 and 4. A further attempt

TABLE V

Corrections to Spectrographic Elements Given by Least-Squares Solutions

	Solution					
	I	II		III		III*a*
	Corr.	Corr.	P.E.	Corr.	P.E.	Corr.
γ, km/sec.	− 1.6	−0.9	± 3.2	+1.5	± 3.7	−1.9
K, km/sec.	−10.1	+3.2	2.6	+0.9	3.1	+5.4
e	− 0.039	+0.052	0.022	+0.010	0.022	+0.058
ω	+ 4°.5	+1°.8	3°.4	−1°.6	3°.3	+3°.2
T, days	− 0.050	+0.022	0.017	−0.009	0.014	−0.003
Normal place of wt. 1, km/sec.	....	....	3.6	....	3.7	....
Single plate of wt. 1, km/sec.	....	....	±12.4	....	±13.0	....

was made to reduce the probable errors (cols. 5 and 6), which, as was expected, was ineffective, but the orbital elements resulting from this solution III were adopted as final.

[1] *Publications of the Allegheny Observatory*, 1, 33, 1908.

The tabulated period is Campbell's for the spectrographic epoch. The eccentricity is the highest for any eclipsing binary, and is even very high for a spectroscopic binary, for which it would normally be associated with a period at least ten times as long. The velocity-curve resulting from these elements is the upper curve in Figure 1, the individual observations being represented by solid squares and the normal places by open circles the radii of which are equal to the

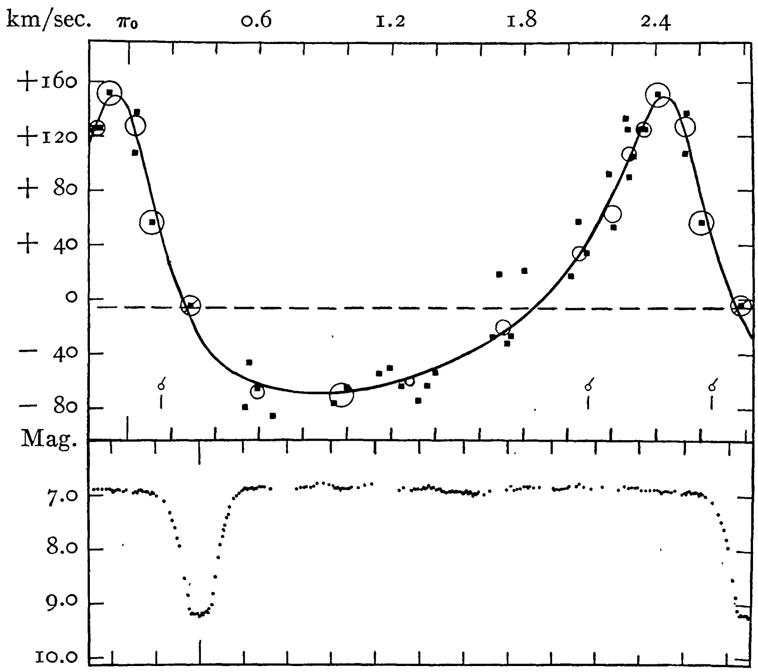

FIG. 1.—*Above:* Velocity-curve of U Cephei. Solid squares represent the individua observations, circles the normal places with their radii equal to the probable errors The barred circle marks the Mount Wilson normal place. Abscissae represent phases from periastron. The computed phases of conjunction are indicated by the usual symbol. *Below:* The light-curve as defined by Dugan's normal places. Abscissae indicate phases from primary minimum. The two curves are adjusted to show their observed relative phases.

weighted probable errors. The Mount Wilson observation is plotted as a barred circle. Residuals from this curve for the individual observations and for the normal places are shown in the last columns of Tables I and II.

The propriety of including in these solutions Joy's velocity of the secondary star at primary minimum is perhaps open to question.

The velocity of a component at conjunction differs from the systemic velocity by $Ke \cos \omega$, which vanishes in general only in cases of circular orbits. In this case, however, on a reasonable assumption for the relative mass of the secondary star, this term is not greatly in excess of the uncertainty of a normal place of its weight, and in view of the lack of observations near this phase, occasioned by the faintness of the secondary, the retention of the observation seemed justified. In any event, its residual is not large, but to test more thoroughly its effect, a solution based on orbit II was carried out, by the use of the same equations of condition as for III with the omission of No. 12. This solution, III*a* of Table V, is thus directly comparable

TABLE VI

FINAL ELEMENTS

Epoch = 1924.5	$\omega = 25\overset{\circ}{.}0 \pm 3\overset{\circ}{.}3$
$P = 2\overset{d}{.}4929507$	$T =$ J.D. $2423966.644 \pm 0\overset{d}{.}014$
$\gamma = -6.0 \pm 3.7$ km/sec.	$a \sin i = 3{,}320{,}000$ km
$K = 109.9 \pm 3.1$ km/sec.	$\dfrac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0.235$
$e = 0.474 \pm 0.022$	

with III. No marked change results, but the eccentricity and periastron *minus* node are both somewhat increased, a result which might have been anticipated from an inspection of the velocity-curve, this observation obviously tending to reduce slightly the asymmetry of the curve. Hence the remarks appearing below concerning orbit III apply with at least equal force to an orbit based wholly upon my plates.

THE PHOTOMETRIC DATA

Photometric orbits of U Cephei, representing a large faint star totally eclipsing a small bright star at primary minimum, have been derived by H. Shapley,[1] R. S. Dugan,[2] and R. H. Baker.[3] Shapley used 695 photometric observations by O. C. Wendell,[4] Dugan, 14,112 measures of his own with a polarizing photometer; and Baker, 305 extra-focal photographs made by Miss Cummings and himself.

[1] *Princeton Contributions*, No. 3; *Astrophysical Journal*, **36**, 269, 1912.

[2] *Princeton Contributions*, No. 5, 1920.

[3] *Laws Observatory Bulletin*, No. 30, 1921.

[4] *Harvard Annals*, **69**, 58, 1909.

The light-curves are in essential agreement, except for the greater depth of Baker's primary minimum occasioned by the late type of the secondary. Dugan's curve, as the most complete, is reproduced in Figure 1 in its observed phase with the velocity-curve above it. All three light-curves are characterized by distinct asymmetry during the primary minimum, in the sense of a more rapid rise than descent. There is some evidence for a slight retardation of the secondary minimum from the mid-phase of the light-curve. Shapley estimates an uncertain tenth-of-a-day retardation; Baker seems to regard it as fairly certain; and Dugan admits its possibility, though in the last two cases the curve is not so well covered at emergence

TABLE VII

THE PHOTOMETRIC ORBITS

	Shapley	Baker	Dugan
Epoch of observation	1902.7	1915.5	1915.5
Kind of solution	Uniform	Uniform	One-third darkened
Eccentricity (assumed)	0	0	0
Lag of secondary minimum from mid-phase	0^d1:	0^d1	Slight
Duration of eclipse	.442	.414	0^d420
Duration of total phase	0.0894	0.101	0.0797
Inclination of orbit	90°	90°	86°4
Ratio of radii, k	0.63	0.60	0.62
Semi-major axis of bright star, r_1	.324	.320	.322
Semi-major axis of faint star, r_2	0.205	0.190	0.200

from the secondary eclipse as it is elsewhere. In the opinion of these computers this lag perhaps indicates some slight orbital eccentricity, but since its effect cannot be separated from the effects of orbital orientation, on account of the shallowness of the secondary minimum, in deriving their orbits, an eccentricity of zero is assumed. Table VII presents the pertinent data of these orbits.

Apart from the question of the real shape of the orbit, there are two discrepancies between the results from the spectrographic and photometric observations: (*a*) the spectrographic elements predict superior conjunction $0^d164 = 3^h9$ earlier than the observed primary eclipse, and (*b*) in consequence of the shape and orientation of the orbit, the spectrographic phase of inferior conjunction occurs $0^d51 = 12^h2$ after the mid-phase occupied by the observed secondary minimum.

ATTEMPTS TO RECONCILE THE DATA

These inconsistencies lead to the question of the reliability of the orbits which are compared, and, in view of the independent confirmation of the photometric results, suspicion falls upon the spectrographic orbit. It is largely for this reason that this paper has been thus delayed, in the expectation of repeating the spectrographic observations at a different epoch at this observatory, but equipment not developing in this direction as it was hoped, this plan has now to be foregone. However, in justification of the spectrographic results it may be pointed out that in the instruments and reduction there was no departure from standard equipment or procedure, that the large range in velocity and the combination of results of widely separated dates into the same normal preclude any likelihood of systematic error, that the observations themselves are thoroughly consistent, showing only moderate relative dispersion, and that Joy's independent velocity behaves quite as would be expected. It is nevertheless possible that many more spectrograms could change the velocity-curve so that the four-hour discrepancy at primary eclipse would be eliminated, although it seems too much to expect that the general form of the curve could be materially altered. Removal of the discrepancy by introducing it into a least-squares solution is unsatisfactory by reason of the large resulting residuals which enter into the equations of conditions, but if the three elements primarily concerned are each changed in the optimum direction (i.e., T increased and ω and e decreased) by four times their probable errors, the time of primary eclipse is satisfactorily predicted. The curve thus resulting from these so arbitrarily adjusted elements,

$$T = \text{J.D. } 2423966.700 \, ,$$

$$\omega = 11\overset{\circ}{.}8 \, ,$$

$$e = 0.386 \, ,$$

does not, however, fit the observations well, running 30–40 km/sec. too high up to a phase of one day from periastron and consistently about 20 km/sec. too low thereafter, while owing to the greater asymmetry of orientation of the orbit the secondary eclipse is worse represented than before by fully two hours. And it is doubtful if there is any virtue in correcting the discrepancy in one eclipse if the other is not improved.

Since the photometric and spectrographic observations here discussed are separated by an interval of nine years, it is tempting to try to reconcile the two orbits on the basis of a rotation of the line of apsides, which must be consequent upon the polar flattening which is usually found in eclipsing systems of short period. The photometric observations of 1915 are then supposed to take place with the major axis of the orbit nearly coincident with the line of sight. Since the eccentricity of U Cephei is too great to be expressed manageably in series form, Russell's usual formulae for the photometric elements of an eclipsing binary with eccentric orbit[1] cannot be used. Instead, the variable radius vector may be introduced into the fundamental equations for circular orbits.[2] If the semi-major axis of the relative orbit is taken as unity, then the distance between centers of the stars is given by

$$\delta^2 = R^2 \cos^2 i + R^2 \sin^2 i \sin^2 \theta , \tag{1}$$

where R is the radius vector and θ has the same meaning that Russell gives it but does not vary uniformly with time. Then

$$R^2 \cos^2 i + R^2 \sin^2 i \sin^2 \theta = r_1^2 \{\varphi(k, \alpha)\}^2 \tag{2}$$

and, following Russell,

$$\frac{(R_1^2 - R_2^2) \cos^2 i + (R_1^2 \sin^2 \theta_1 - R_2^2 \sin^2 \theta_2) \sin^2 i}{(R_2^2 - R_3^2) \cos^2 i + (R_2^2 \sin^2 \theta_2 - R_3^2 \sin^2 \theta_3) \sin^2 i} = \psi(k, \alpha_1) . \tag{3}$$

Unless the principal eclipse takes place at periastron under rather special circumstances of the stellar radii, a system having the light-curve of U Cephei must have $i = 90^\circ$ nearly, so that as a first approximation, to be tested later, we may write

$$\psi(k, \alpha_1) = \frac{R_1^2 \sin^2 \theta_1 - R_2^2 \sin^2 \theta_2}{R_2^2 \sin^2 \theta_2 - R_3^2 \sin^2 \theta_3} \tag{4}$$

or

$$R_1^2 \sin^2 \theta_1 = A + B\psi(k, \alpha_1) , \tag{5}$$

[1] *Astrophysical Journal*, **36**, 54, 1912. [2] *Ibid.*, **35**, 315, 1912.

where the R's are now included in A and B. If now the values for θ are computed from the adopted spectrographic orbit, the usual tables for circular orbits may be used for finding the ratio of the radii of the stars, k. If equation (2) is then written for θ_6 and θ_9 and the ratio taken, a little recombination of terms yields an expression for $\cos^2 i$, and r_1 may now be determined from (2) written for θ_6 or for θ_9. Finally, the light-curve may be represented by (5); but, since there are now two variables, R and θ, the former varying the more slowly, it will usually be necessary to proceed by successive approximations, especially at some distance from apastron.

Using this method, but without taking the trouble to rectify the light-curve, especially since this becomes very complex and highly

TABLE VIII

PHOTOMETRIC ELEMENTS FOR PRIMARY ECLIPSE AT APASTRON

Speed of orbital rotation	$=13°$ per year
	$r_1=0.198$*
	$r_2=\ .133$*
	$k=0.67$
	$i=90\overset{\circ}{.}0$
Ratio of surface brightness	$=15.8$

* Semi-major axis of relative orbit = unity.

uncertain with a large eccentricity, solutions were made on the assumption of primary eclipse taking place at periastron and at apastron. The former assumption must at once be ruled out, because the resulting stellar radii are so great as to cause interpenetration at periastron,[1] but the second assumption leads to apparently reasonable elements, as shown in Table VIII.

The ratio of surface brightness is in good agreement with the spectral types. At periastron the stellar surfaces are separated by a distance approximately equal to the radius of the larger star. Except for the asymmetry, which was not explicitly introduced, the primary light-minimum is well represented, but the shallow secondary minimum, while well represented as to depth, has a duration rather less than half the observed. Apart from this last point, which is some-

[1] Apart from the difficulties of phase, the spectrographic elements of orbit III also result in a geometrically impossible representation of the observed primary minimum.

what involved in observational difficulties, there are four well-known tests of orbital rotation, to which the observations of U Cephei do not respond favorably: (*a*) The duration of eclipses should be alternately long and short as they occur alternately near apastron and periastron. (*b*) The occurrence of secondary eclipse with respect to primary should oscillate about the mid-phase. (*c*) The apparent photometric period should undergo a regular oscillation about a mean. (*d*) The asymmetry of light-minimum should alternately involve the descending and ascending branches in opposite sense, but since other asymmetrical characteristics might possibly effectively mask this it may be dismissed as of secondary importance.

TABLE IX

DURATION OF PRIMARY ECLIPSE PHASES BELOW VARIOUS MAGNITUDES AT FOUR EPOCHS

	MAGNITUDE				
	7.0	7.5	8.0	8.5	9.0
Pickering,* 1880–1881...	$0^d.33$	$0^d.23$	$0^d.18$	$0^d.16$	$0^d.12$
Pickering, 1895–1897....	.40	.24	.18	.13	.09
Wendell, 1895–1902.....	.47	.26	.18	.15	.12
Dugan, 1914–1916.......	0.37	0.23	0.17	0.13	0.11

* Data for the first three entries are taken from Müller and Hartwig, *Geschichte und Literatur des Lichtwechels*, 1, 28, 1918; for the last, from Dugan's paper to which reference has been made.

a) *The duration of eclipses.*—Mrs. Shapley, in her discussion of the period of U Cephei,[1] shows that Wendell's observed times of the star passing magnitude 8.4 on the descending and ascending branches indicate very effectively that there was no change in the duration of primary eclipse at the corresponding phase in the interval from 1896 to 1911. Further confirmation of this extending from 1880 to 1916 is contained in Table IX, which gives the duration of five phases of primary minimum at four epochs. The intermediate magnitudes deserve greatest weight, since in passing through these the brightness is varying most rapidly. Clearly the duration of eclipse is substantially constant over an interval of thirty-six years, whereas an eccentricity of the order indicated by the spectrographic data implies at least twice the duration of eclipse at apastron as at periastron.

[1] *Astrophysical Journal*, 44, 51, 1916.

b) *The phase of secondary eclipse.*—Even if the apparent photometric period appears disturbed by other causes, there should at least result from orbital rotation a periodic displacement of secondary minimum about mid-phase by an amplitude indicated in this case to be at least fifteen hours. Evidence available to me on this point is negative, the three complete light-curves discussed above showing the secondary minimum close to mid-phase, but this of itself is not very conclusive, since the interval between Wendell's epoch and Dugan's and Baker's epoch is favorable to a rotation of 180° at a rate approximating 13° per year.

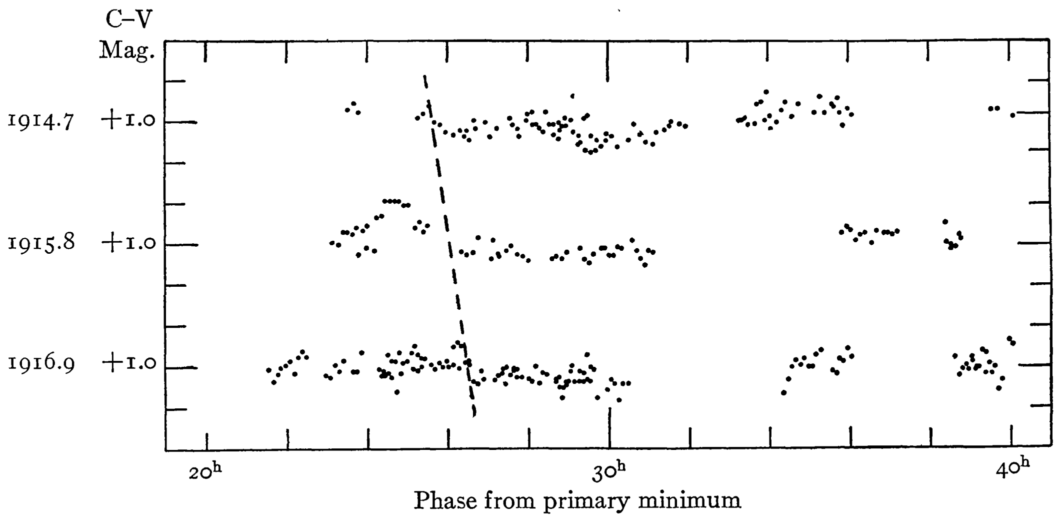

FIG. 2.—Dugan's unweighted individual observations through secondary minimum at three epochs. The ruled magnitude interval=0.2 mag.

Perturbations of this magnitude ought to show in Dugan's observations of secondary minimum, which cover a space of more than two years. Figure 2 shows all of his individual observations between phases of twenty and forty hours from primary minimum plotted in three groups whose mean dates are separated by about a year. Unfortunately, the emergence is not so consistently well observed as the immergence, so that only the time of beginning of eclipse is useful here. Evidence for a progressive retardation of phase, which is in order at this epoch under the circumstances in question, is pretty weak, and in any case the shift can hardly be greater than that indicated by the inclined broken line—much too small for a uniform orbital rotation of the speed suggested.

c) *The period.*—Mrs. Shapley[1] has very completely assembled the observations upon the primary minimum of U Cephei up to 1914, and computed their residuals from Wendell's light-elements, concluding that, apart from a rather abrupt change in the period in 1905, the changes, while real, were slight and complex. In Table X are gathered residuals from observations made since Mrs. Shapley's paper, and an earlier one by Schwerd.[2] All data are plotted in Figure 3.

In a rotating orbit of slight eccentricity a plot of residuals from linear elements against time gives a sine-curve,[3] but when, as in this

TABLE X

RESIDUALS FROM WENDELL'S ELEMENTS

Epoch J.D.	O−C	Observer	Reference
2388855	$0^d.733$:	Schwerd	Chandler, *A.J.*, **9**, 49
2421290	.0229	Dugan	*Princeton Contr.*, No. 5
2421997	.0275	Campbell	Personal communication
2422337	.0297	Dugan	*Princeton Contr.*, No. 5
2423054	.040	Campbell	*H.B.*, No. 762, 1922
2423249	.047	Campbell	Personal communication
2423727	.065	Stetson	Personal communication
2424804	.0846	Campbell	*H.B.*, No. 842, 1927
2425557	.0975	Campbell	*Ibid.*, No. 862, 1928
2425901	0.101	Campbell	*Ibid.*, No. 871, 1929

case, the eccentricity is much too large to be expanded usefully into a series, the resulting curve, having an amplitude here of eight or ten hours, is quite asymmetrical, although the actual curve is hardly computable, since, on account of the variation of the perturbative force throughout the orbit, the orbital rotation will not be uniform. It is at first tempting to interpret the plotted curve as a part of one resulting from orbital rotation, though the period would probably not fit well. A uniformly increasing period, however, gives a residual curve of the form of a vertical parabola, so that it appears more informative to deduce the true periods from the figure by computing

[1] *Ibid.*

[2] For a discussion of the justification of using Schwerd's observation see Dugan, *op. cit.*, p. 29.

[3] André, *Traité d'astronomie stellaire*, **2**, 253, Paris, 1900.

from the slope at any epoch the corresponding correction to Wendell's period. Figure 4, so derived, which is essentially an extension of Dugan's to later dates, shows that the complete history of the period, as far back as our observations extend, involves an essentially uniform increase with small superposed oscillations. With due

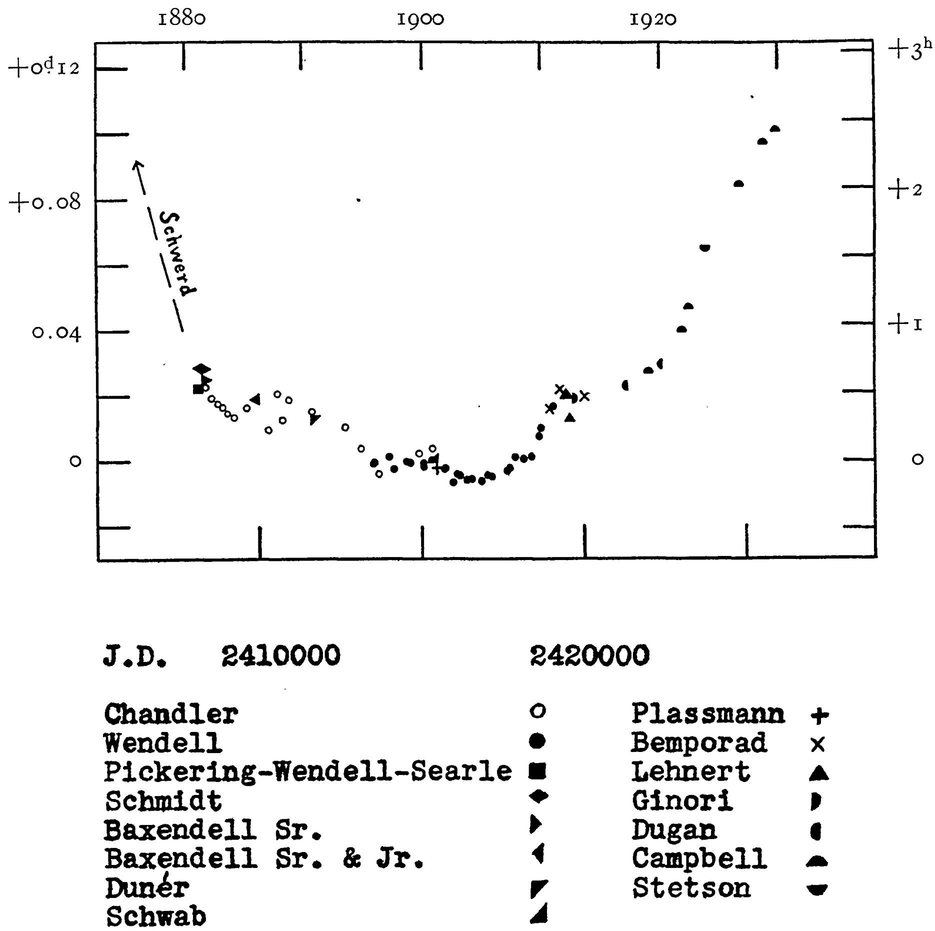

FIG. 3.—Residuals from Wendell's elements. As far as the hump about 1912 the curve is essentially the same as Mrs. Shapley's. The various observers are indicated by the symbols above. For the original references see Mrs. Shapley's and Dugan's papers, and Table X of this paper.

consideration to the inherent inaccuracies of graphical differentiation, it seems difficult to doubt the reality of those oscillations, since the corresponding oscillations in the residual curve from which they are derived are well above the presumable minute or two uncertainties[1] of observation. But their amplitude and period, especially the former, are much too small to fit any hypothesis of advance of periastron.

The definite reconciliation of the photometric and spectrographic data on the basis of a rotation of the orbit in its own plane, which was tentatively suggested as a possibility in a preliminary note,[2] has then to be abandoned in spite of the favorable representation of the light-curve on the assumption of a reasonable speed of rotation. It then becomes equally difficult to explain the apparent absence of such rotation in a system presumably so well suited to its detection.

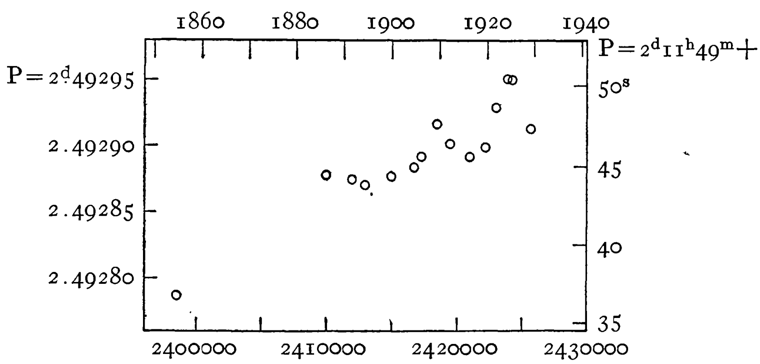

FIG. 4.—The period of U Cephei through eighty years

U Cephei appears to be a system much more complex than its idealized light-curve would imply. Granted Dugan's reasonable explanation of the uniform increase of period as due to tidal evolution, the oscillations of the period are still a mystery. G. Viola[3] recently, from mean light-curves made some years ago at Catania and Capodimonte, and H. T. Stetson[4] earlier from single light-curves, have shown a lack of constant light during the total phase of primary minimum. Unfortunately, the secondary minimum is so shallow as to be studied only with considerable difficulty. Probably the most effective line of attack upon these complexities at the present time would be close following spectrographically.

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August 1930

[1] Dugan, *op. cit.*, p. 33.

[2] *Popular Astronomy*, **38**, 401, 1930.

[3] *Atti Lincei* (6th ser.), **10**, 508.

[4] *Astrophysical Journal*, **43**, 325, 1916; *Popular Astronomy*, **32**, 623, 1924.