

## Ritz-type variable speed of light (VSL) cosmology

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**Abstract:** This article develops a variable-speed-of-light (VSL) cosmology model. VSL has been used previously in cosmology models, in which either (i) the physical constants vary over time or (ii) the Lorentz invariance is broken locally. VSL is also allowed in the relative-velocity-based approach, which is used in the current article to propose a VSL-type cosmology model. Additionally, the article evaluates the model's potential to match current cosmological observations. © 2014 Physics Essays Publication. [<http://dx.doi.org/10.4006/0836-1398-27.4.523>]

**Résumé:** Cet article développe un modèle cosmologique basé sur une vitesse de la lumière variable (VSL). La VSL a été utilisée auparavant par des modèles cosmologiques, dans lesquels (i) les constantes physiques varient au cours du temps, ou (ii) l'invariance de Lorentz est localement violée. La VSL est également autorisée dans l'approche basée sur une vitesse relative, qui est utilisée dans le présent article pour proposer un modèle cosmologique de type VSL. En outre, cet article examine le potentiel du modèle pour égaler les observations cosmologiques actuelles.

**Key words:** Cosmology; Ritz; Time Dilation; Farther Dimmer; Superluminal Ejecta; VSL.

### I. INTRODUCTION

The large number of anomalies in cosmological observations has led to substantial interest in alternatives to the standard big-bang type cosmology, e.g., Refs. 1–5. The main contribution of this article is to propose a Ritz-type, variable-speed-of-light (VSL) cosmology model where the velocity of light is dependent on source velocity. A recent review of Ritz's approach shows that the astronomical observations that were initially thought to be contradictory were later found to be consistent.<sup>6</sup> It is noted that VSL has been used, previously, in cosmology models,<sup>7</sup> where physical constants (such as the gravitational constant) are allowed to vary over time. The relationships between the temporal variations of the different physical constants can be determined to match physical observations such as relativistic electromagnetism. The current article evaluates the potential for an alternate (Ritz-type VSL) model to match cosmological observations and explain current anomalies.

In Ritz-type models, VSL is assumed wherein the velocity of the source augments the speed of light. Several researchers had suggested investigating astronomical data to test Ritz-type models—such astronomical observations (e.g., irregularities in observations of double-star system) that were initially thought to be contradictory were later found to be consistent.<sup>6</sup> Nevertheless, there remained several challenges such as the inability to (i) explain Fresnel drag and (ii) the lack of an accompanying electromagnetic theory, which would require modification of Maxwell's equations. These issues were recently addressed in Ref. 8, which extends Maxwell's equations to enable a Ritz-type VSL.

There are two main changes—(i) the partial time derivative in Maxwell's equations is replaced by the total time derivative and (ii) the electromagnetic force depends on the relative velocity between particles, which is a modification of Weber's approach.<sup>9</sup> The resulting relative-velocity-based model<sup>8</sup> not only captures relativity effects in optics (such as the Fresnel drag and transverse Doppler effects) but also explains apparent discrepancies between predicted and measured energy in: (i) the absorption of high-energy particles in cloud chambers<sup>10</sup> and (ii) the average energy determination of the X-ray spectrum using magnetic fields.<sup>11</sup> Based on these efforts, the current article proposes a Ritz-type VSL cosmology model.

The article begins by showing (in Sec. II) that the proposed model can be used to derive the standard Hubble law, and is consistent with the time dilation seen in current cosmological observations.<sup>12</sup> Then, periodic photosphere motions are investigated for its effect on VSL cosmology and its ability to match stellar observations in Sec. III. Issues in quasar observations such as the apparent lack of time dilation in quasar light curves,<sup>13</sup> even though time dilation has been observed in supernovae (SNe) light curves,<sup>12</sup> are studied in Sec. IV. The model is used to also investigate, in Sec. IV, observations that (i) link some quasars with low redshift galaxies<sup>14,15</sup> and (ii) indicate the presence of superluminal ejecta.<sup>16</sup> Consistency of the model with recent farther-dimmer relation<sup>17</sup> in SNe observations is shown in Sec. V. The proposed model leads to temporal-and-spatial distortions in cosmological observations—the impact of such distortions is discussed in Sec. VI. Finally, potential large-scale anisotropies, e.g., in the Hubble constant, are discussed in Sec. VII, which is followed by the conclusions section.

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## II. DERIVATION OF THE HUBBLE LAW

### A. The model

The current article considers a big-bang-type model where the universe (containing astrophysical objects) is a uniformly expanding spherical shell (geometry axiom), which is similar to Newtonian cosmology models, e.g., Ref. 18. However, in contrast to the Newtonian cosmology model where the expansion speed increases linearly with the distance from the center,<sup>18</sup> the expansion speed is constant in the current model. Nevertheless, it is shown that even with a constant expansion speed, the Ritz-type VSL model is consistent the farther–dimmer relation seen in recent SNe-based observations.<sup>17</sup>

Consider light emitted in all directions at the standard speed of light  $c$  and frequency  $\nu_e$  by an emitter  $e$  when it is at the location  $e_e$  at time  $t_1$  as shown in Fig. 1. According to the geometry axiom, the emitter is moving at constant speed  $V$ , i.e., velocity  $V_{\underline{I}_e}$  with respect to an inertial frame  $I_a$  at a central position  $a$  as shown in Fig. 1, where  $\underline{I}_e$  is a fixed, unit vector.

After time  $\Delta t_{1,2}$ , let the light reach an observer  $o$  at location  $o_o$  at time  $t_2 = t_1 + \Delta t_{1,2}$ , where the observer is moving at constant velocity  $V_{\underline{I}_o}$  with respect to the inertial frame  $I_a$  and  $\underline{I}_o$  is a fixed, unit vector. With respect to the emitter inertial frame  $I_e$  (moving with the emitter as shown in Fig. 1), during the time interval  $\Delta t_{1,2}$ , light has traveled a distance  $d_o = d(e_o, o_o) = c\Delta t_{1,2}$ , i.e., reached a shell of radius  $d_o$  centered at  $e_o$  at time  $t_2$ .

The speed  $c_o$  of the light observed in an inertial frame  $I_o$  on the observer  $o$  (at location  $o_o$ ) is given by

$$c_o \underline{y} = V_e \underline{I}_e + c \underline{y} - V_o \underline{I}_o = V_{\underline{I}_e} + c \underline{y} - V_{\underline{I}_o}, \quad (1)$$

where  $\underline{y}$  is a fixed unit vector. As in Ritz-type models, the velocity of light  $\underline{c}$  is added to the velocity of the source  $\underline{v}_e$ , i.e.,  $\underline{c} + \underline{v}_e$  to find the propagation velocity (the velocity axiom) in the relative-velocity-based approach. Although the proposed approach is different from the tired-light-type hypothesis, e.g., Ref. 1, the current model also results in a

distance–redshift relation that matches the Hubble law as shown below.

The magnitude  $c_o$  of the observed light velocity can be found using the similar triangles  $\Delta(ae_eo_e)$  and  $\Delta(ae_o o_o)$ , to obtain the following relationship between: (i) the distance  $R = d(a, e_o) = d(a, o_o)$  from the center  $a$  to the observer and emitter at observation time  $t_2$  and (ii)  $d_e = d(e_e, o_e)$  the distance between observer and emitter at the emission time instant  $t_1$

$$\begin{aligned} \frac{d_e}{d_o} &= \frac{c_o \Delta t_{1,2}}{c \Delta t_{1,2}} = \frac{c_o}{c} = \frac{R - V \Delta t_{1,2}}{R} = \frac{R - V(\frac{d_o}{c})}{R} \\ &= 1 - \frac{V}{Rc} d_o, \end{aligned} \quad (2)$$

where  $d_e = c_o \Delta t_{1,2}$  since this initial distance at time  $t_1$  is covered at speed  $c_o$  in the observer frame  $I_o$ . Note that the observed light speed  $c_o$  is less than the standard speed of light  $c$  from Eq. (2) since  $d_e \leq d_o$ , i.e.,

$$c_o \leq c. \quad (3)$$

### B. Time dilation, red shift, and energy reduction

The cosmological expansion and the reduction in observed light speed result in three effects discussed below: (i) perceived time dilation; (ii) redshift; and (iii) energy reduction.

#### 1. Perceived time dilation

The emitter is seen to move away from the observer (in the observer frame of reference  $I_o$ ) at a speed  $V_{e,o}$  described by, from Eq. (1),

$$V_{e,o} \underline{y} = V_{\underline{I}_e} - V_{\underline{I}_o} = (c - c_o) \underline{y}. \quad (4)$$

Consider two photons emitted at time instants  $t_1$  and  $t_1 + t_e$ , which move toward the observer at speed  $c_o$  in the observer frame of reference  $I_o$ . During the time interval  $t_e$ , the distance  $d_e = d(e_e, o_e)$  between the emitter and the observer

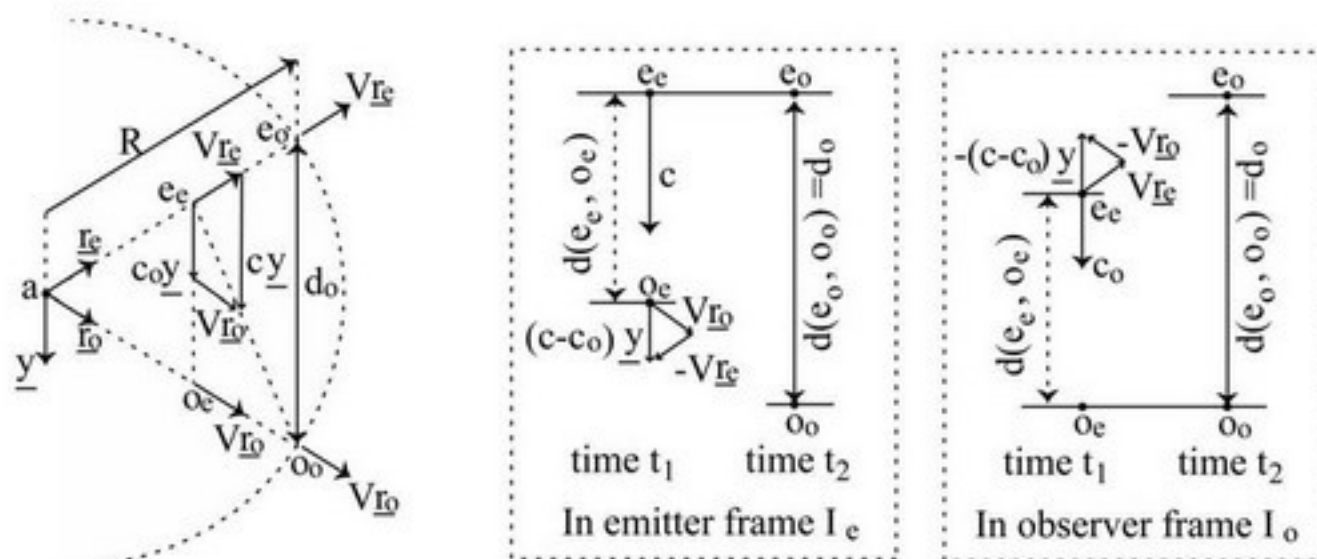


FIG. 1. VSL based on reference frame: (left) reference frame at center of cosmological shell; (middle) reference frame associated with the emitter; and (right) reference frame associated with the observer. Light emitted from point  $e_e$  at time  $t_1$  reaches observer at  $o$  at time  $t_2$ , after time  $\Delta t_{1,2} = t_2 - t_1$ . Distances at observation time  $t_2$  of the emitter and observer from the center  $a$  are  $d(a, e_o) = R$  and  $d(a, o_o) = R$ , respectively. Both emitter and observer are moving away from the center  $a$  at constant speed  $V_e = V_o = V$ . Distance between observer and emitter at emission time  $t_1$  is  $d(e_e, o_e) = d_e$  and at observation time  $t_2$  is  $d(e_o, o_o) = d_o$ .

has increased by  $V_{e,o}t_e$ . Hence the time interval  $t_o$  between the observations of the two photons (in the observer frame  $I_o$ ) is given by, from Eq. (4),

$$t_o = \left( t_e + \frac{d_e + V_{e,o}t_e}{c_o} \right) - \frac{d_e}{c_o} = \left[ t_e + \frac{(c - c_o)t_e}{c_o} \right] = \frac{c}{c_o} t_e. \tag{5}$$

Therefore, a time interval  $t_e$  in the emitter frame appears (optically) as a dilated time interval  $t_o$  in the observer frame with

$$t_o = \frac{c}{c_o} t_e \geq t_e, \tag{6}$$

since  $c \geq c_o$  by Eq. (3). Consequently, events in the emitter frame  $I_e$  appear (optically) to occur at a slower rate in the observer frame  $I_o$  leading to a perceived time dilation.

### 2. Redshift

The observed frequency  $\nu_o$  is reduced with respect to the emitted frequency  $\nu_e$  due to the Doppler effect. For example,  $N_\nu = \nu_e t_e$  pulses sent at frequency  $\nu_e$  in time  $t_e$  from the emitter are received at the observer in time  $t_o$ , as in Eq. (6). Therefore, the observed frequency  $\nu_o$  is, by using Eq. (6)

$$\nu_o = \frac{N_\nu}{t_o} = \frac{t_e}{t_o} \nu_e = \frac{c_o}{c} \nu_e, \tag{7}$$

which corresponds to a redshift  $z \geq 0$  given by

$$z = \frac{\nu_e - \nu_o}{\nu_o} = \frac{\nu_e}{\nu_o} - 1 = \frac{c}{c_o} - 1, \tag{8}$$

that can be rewritten as

$$1 + z = \frac{c}{c_o}. \tag{9}$$

Moreover, the distance  $d_o$  in Eq. (2) can be rewritten in terms of the redshift  $z$  as

$$d_o = \frac{Rc}{V} \left( 1 - \frac{c_o}{c} \right) = \frac{Rc}{V} \left( 1 - \frac{1}{1+z} \right) = \frac{Rc}{V} \left( \frac{z}{1+z} \right), \tag{10}$$

and the observed frequency in Eq. (7) can be rewritten as

$$\nu_o = \frac{c_o}{c} \nu_e = \frac{1}{1+z} \nu_e. \tag{11}$$

Note that the time dilation expression can be written in terms of the red shift from Eqs. (6) and (9) as

$$t_o = \left( \frac{c}{c_o} \right) t_e = (1+z)t_e. \tag{12}$$

The time dilation of  $1+z$  is consistent with cosmological observations such as the time broadening of SNe light curves.<sup>12</sup>

### 3. Energy reduction

The energy of a photon is reduced in the observer inertial frame  $I_o$  when compared to the emitter inertial frame  $I_e$  due to the reduction in the photon frequency between the two frames, from  $\nu_e$  to  $\nu_o$  given by Eq. (11). The ratio of perceived energy  $E_o$  (of photons) in the observer frame to the energy  $E_e$  (of the corresponding photons) in the emitter frame is given by

$$\frac{E_o}{E_e} = \frac{h\nu_o}{h\nu_e} = \frac{1}{1+z}, \tag{13}$$

where  $h$  is the Planck constant. Note that in addition to the change in the observed energy of the light-quanta due to a reduction of the observed frequency, as in Eq. (13), the observed energy is reduced by the perceived time dilation, as in Eq. (12), since it leads to a change in the arrival rate of light-quanta. Hubble suggested both these effects as corrections in Ref. 19.

### C. Luminosity distance and red shift

Let  $L_e$  be the total energy of photons emitted from  $e_e$  per unit time. Then, the energy  $L_e \Delta t_e$  emitted in a small time interval  $\Delta t_e$  is spread over a shell of radius  $d_o$  centered at point  $e_o$  after time  $\Delta t_{1,2}$  as in the middle plot of Fig. 1. The resulting energy per unit surface area  $E_e$  of this shell (in the emitter inertial frame  $I_e$ ) is given by, from Eq. (10),

$$E_e = \frac{L_e}{4\pi d_o^2} \Delta t_e = \frac{L_e}{4\pi \left( \frac{Rc}{V} \right)^2 \left( \frac{z}{1+z} \right)^2} \Delta t_e. \tag{14}$$

In this same situation, the energy per unit surface area  $E_o$  observed at  $o_2$  in the dilated-time interval  $\Delta t_o$  (in the observer inertial frame  $I_o$ ) is, from Eqs. (12)–(14),

$$E_o = \frac{E_e}{1+z} = \frac{L_e}{4\pi \left( \frac{Rc}{V} \right)^2 \left( \frac{z^2}{1+z} \right)} \Delta t_e = \frac{L_e}{4\pi \left( \frac{Rc}{V} \right)^2 \left( \frac{z^2}{1+z} \right)^{1+z}} \frac{\Delta t_o}{1+z} = \frac{L_e \Delta t_o}{4\pi \left( \frac{Rc}{V} z \right)^2}. \tag{15}$$

Then, the observed brightness  $B_o$  (i.e., energy per unit area per unit time in the observer inertial frame  $I_o$  at  $O_o$ ) is given by

$$B_o = \frac{E_o}{\Delta t_o} = \frac{L_e}{4\pi \left( \frac{Rc}{V} z \right)^2} = \frac{L_e}{4\pi (d_L)^2}, \tag{16}$$

where the observed luminosity distance  $d_L$  increases linearly with the redshift  $z$  as

$$d_L = \frac{R}{V} cz = (H^{-1}) cz, \tag{17}$$

which can be rewritten as (the Hubble law)

$$V_{\text{apparent}} = cz = Hd_L, \quad (18)$$

with  $V_{\text{apparent}}$  being the apparent speed away from the observer (based on the redshift  $z$ ) and

$$H = \frac{V}{R}, \quad (19)$$

is the Hubble constant.

#### D. Relation between the different distances

The model yields the expected relation between angular distance, proper distance and the observed luminosity distance  $d_L$ , as described below. In the observer inertial frame  $I_o$ , the emission is initiated at distance  $d_e$ —although the emitter is then seen to move away at speed  $(c - c_o)$ . Therefore, the angle  $\theta_e$  of the emitter in the sky and the perceived size  $S_e$  are related to the angular distance  $d_e$  as  $S_e = \theta_e d_e$ . The distance  $d_o$  between the emitter and observer at the observation time is considered as the proper distance at the time of observation. The luminosity distance  $d_L$  and the proper distance  $d_o$  are related from Eqs. (10) and (17),

$$d_o = \frac{Rc}{V} z \left( \frac{1}{1+z} \right) = \frac{d_L}{1+z}. \quad (20)$$

The angular distance  $d_e$  and the proper distance  $d_o$  are related from Eqs. (2) and (9),

$$d_e = \frac{c_o}{c} d_o = \frac{d_o}{1+z}, \quad (21)$$

resulting in the following relation between the luminosity distance  $d_L$ , the proper distance  $d_o$ , and the angular distance  $d_e$ :

$$d_L = (1+z)d_o = (1+z)^2 d_o. \quad (22)$$

### III. EFFECT OF VSL ON STELLAR OBSERVATIONS

The time dilation is affected by the speed and radius of the photosphere; the associated distortion of the light curves from nearby stars is studied in this section.

#### A. Periodic photosphere motion

Consider a nearby star at distance  $d_{e,o}$ , for which cosmological expansion effects (such as time dilation) are negligible. This allows the following analysis to focus on just the effect of the velocity addition on the speed of light. Let the motion of the photosphere be periodic and continuous, and consider one time period, i.e., time interval  $[t_{e,i}, t_{e,i} + T_p]$ , where  $T_p$  is time period of the photosphere motion. For any time instant  $t = t_{e,i} + t_e \in [t_{e,i}, t_{e,i} + T_p]$ , i.e., for the shifted emission time  $t_e = (t - t_{e,i}) \in [0, T_p]$ , let the acceleration  $a_p(t_e) = c\dot{\beta}_p(t_e)$  of the photosphere be given by  $\dot{\beta}_p(t_e) = f_p(t_e) + C_a$  where  $f_p(t_e)$  is a periodic function with time period  $T_p$  in the emitter frame  $I_e$ . Note that the functions are described in terms of the shifted, emission time  $t_e$ . The constant  $C_a$  is chosen such that the speed  $v_p = c\beta_p$  of the

photosphere is periodic, and continuous at the endpoints of the time interval  $t_e \in [0, T_p]$ , e.g.,

$$\begin{aligned} \frac{v_p(T_p)}{c} - \frac{v_p(0)}{c} &= \beta_p(T_p) - \beta_p(0) = \int_0^{T_p} \dot{\beta}_p(t_e) dt_e \\ &= T_p C_a + \int_0^{T_p} f_p(t_e) dt_e = 0. \end{aligned} \quad (23)$$

Moreover, the initial velocity  $v_p(0)$  is chosen to ensure that the radial photosphere position  $r_p(t_e)$  is periodic, and continuous at the endpoints of the time interval  $t_e \in [0, T_p]$ , e.g.,

$$\frac{r_p(T_p)}{c} - \frac{r_p(0)}{c} = \int_0^{T_p} \beta_p(t_e) dt_e = 0. \quad (24)$$

An example photosphere trajectory is shown in Fig. 2.

#### B. Observation and emission time intervals

Consider two photons emitted at time instants  $t = t_{e,i}$  and  $t = t_{e,i} + t_e$ , which move toward the observer at speeds  $c + c\beta_p(0)$  and  $c + c\beta_p(t_e)$ , respectively, in the emitter frame  $I_e$ . Let the photons reach an observer  $o$  (at a distance  $d_{e,o}$ ) at the two time instants  $t = t_{o,i}$  and  $t = t_{o,i} + t_o$ , respectively, where

$$t_{o,i} = t_{e,i} + \frac{d_{e,o} - r_p(0)}{c + c\beta_p(0)}, \quad (25)$$

$$t_{o,i} + t_o = t_{e,i} + t_e + \frac{d_{e,o} - r_p(t_e)}{c + c\beta_p(t_e)}, \quad (26)$$

and the shifted observer time  $t_o = (t - t_{o,i})$  can be found by subtracting the above two expressions as

$$\begin{aligned} t_o &= t_e + \frac{d_{e,o}}{c} \left[ \frac{1}{1 + \beta_p(t_e)} - \frac{1}{1 + \beta_p(0)} \right] \\ &\quad - \frac{1}{c} \left[ \frac{r_p(t_e)}{1 + \beta_p(t_e)} - \frac{r_p(0)}{1 + \beta_p(0)} \right] \\ &= t_e - \frac{d_{e,o}}{c} \frac{\beta_p(t_e) - \beta_p(0)}{[1 + \beta_p(t_e)][1 + \beta_p(0)]} \\ &\quad - \frac{1}{c} \left[ \frac{r_p(t_e)}{1 + \beta_p(t_e)} - \frac{r_p(0)}{1 + \beta_p(0)} \right]. \end{aligned} \quad (27)$$

For periodic emissions (with period  $T_p$ , which is assumed to be large compared to the time for light to travel across the photosphere radius, i.e.,  $r_p/c$ ) and for small photosphere speeds (i.e., small  $\beta_p$ ), Eq. (27) can be simplified to

$$\begin{aligned} t_o &\approx t_e - T_p \left( \frac{\beta_{p,\text{max}} d_{e,o}}{c T_p} \right) \left[ \frac{\beta_p(t_e) - \beta_p(0)}{\beta_{p,\text{max}}} \right], \\ &= t_e - T_p \Gamma_p \left[ \frac{\beta_p(t_e) - \beta_p(0)}{\beta_{p,\text{max}}} \right], \end{aligned} \quad (28)$$

where  $c\beta_{p,\text{max}}$  corresponds to the maximum absolute value of the photosphere speed. Without photosphere motion, the

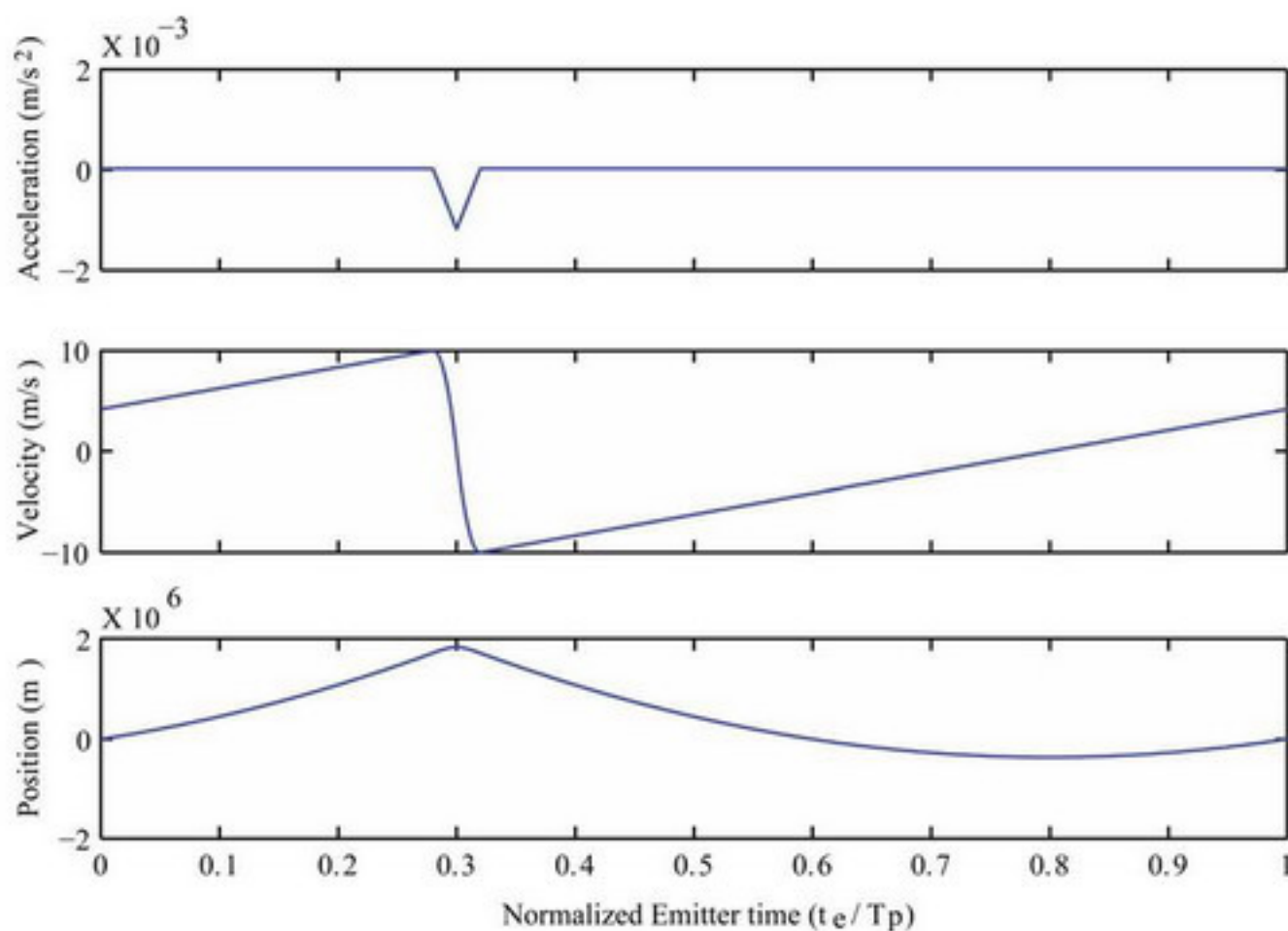


FIG. 2. (Color online) Example periodic photosphere motion for emitter time  $t_e \in [0, T_p]$ . The acceleration varies linearly in the normalized time intervals  $t_e/T_p \in [0.28, 0.3]$  and  $t_e/T_p \in [0.3, 0.32]$ , and is constant elsewhere. Specific values of the acceleration are  $a_p = 2.4 \times 10^5 \text{ m/s}^2$  at  $t_e/T_p = 0$  and  $a_p = -1.2 \times 10^5 \text{ m/s}^2$  at  $t_e/T_p = 0.3$ .

observation and emission time intervals would be equal, i.e.,  $t_o = t_e$ . However, for the same emission time interval  $t_e$ , the observation time interval  $t_o$  is smaller if the photosphere speed,  $\beta_p$  is larger since it takes less time for the emission to travel the distance  $d_{e,o}$ .

For example, the observation and emission time intervals  $t_o$  and  $t_e$ , as in Eq. (28), are compared in Fig. 3 for the photosphere motion in Fig. 2. The parameter  $\Gamma_p$  in Eq. (28) is chosen to be  $\Gamma_p = 4 \times 10^{-4}$ , which corresponds to a time period  $T_p$  of 10 days, distance  $d$  of 100 parsecs, and maximum (absolute) photosphere speed of  $v_p = 10 \text{ m/s}$ . Note that other combinations of these terms can also lead to the same parameter  $\Gamma_p$  value. The observation and emission time intervals  $t_o$  and  $t_e$  are similar. Therefore, for clarity, one of the curves ( $t_o/T_p$ ) is displaced upward in the top plot of Fig. 3. Moreover, the difference  $(t_o - t_e)/T_p$  is shown in the bottom plot in Fig. 3.

### C. Brightness variation follows photosphere acceleration

If the energy-emission rate from the star were constant, then without photosphere motion, the observed brightness  $B_o$  would be uniform, i.e.,  $B_o(t_o) = B^*$ . However, with photosphere motion, photons that are emitted in evenly spaced intervals of time will not be observed in evenly spaced intervals of time due to changes in the time  $t_o$  between observations as quantified in Eq. (28). The variation in the observed brightness is numerically evaluated by discretizing the emission times into small intervals, and mapping the emitted photons into discretized observation time intervals. The resulting observed brightness  $B_o$ , with and without photosphere motion, is compared in Fig. 4.

With a small photosphere motion, the variation of the observed brightness  $B_o$  (in Fig. 4) has a similar trend as the photosphere acceleration  $a_p$  (in Fig. 2). To clarify this, photons observed during a small time interval  $\Delta t_o$  around the shifted, observation time  $t_o$  could be related to those from the associated emission time interval  $\Delta t_e$ , from Eq. (28),

$$\begin{aligned} \Delta t_o &= \left[ \frac{dt_o}{dt_e}(t_e) \right] \Delta t_e = \left[ 1 - \left( \frac{\Gamma_p T_p}{\beta_{p,\max}} \right) \frac{d\beta_p}{dt_e}(t_e) \right] \Delta t_e, \\ &= \left[ 1 - \left( \frac{\Gamma_p T_p}{c \beta_{p,\max}} \right) a_p(t_e) \right] \Delta t_e \\ &= \left[ 1 - \left( \frac{d_{e,o}}{c^2} \right) a_p(t_e) \right] \Delta t_e. \end{aligned} \quad (29)$$

Hence, the observed brightness, with photosphere motion, is related to the observed brightness  $B^*$  without photosphere motion through

$$B_o(t_o) = B^* \frac{\Delta t_e}{\Delta t_o} = B^* \frac{1}{1 - \left( \frac{d_{e,o}}{c^2} \right) a_p(t_e)}, \quad (30)$$

where the shifted observation and emission times  $t_o$  and  $t_e$  are related by Eq. (28). In the logarithmic scale, for sufficiently small photosphere motions, i.e., sufficiently small acceleration  $a_p$ ,

$$\begin{aligned} \log_{10}[B_o(t_o)] &= \log_{10}[B^*] - \log_{10} \left[ 1 - \left( \frac{d_{e,o}}{c^2} \right) a_p(t_e) \right], \\ &\approx \log_{10}[B^*] + \left[ \frac{1}{\ln(10)} \left( \frac{d_{e,o}}{c^2} \right) \right] a_p(t_e), \end{aligned} \quad (31)$$

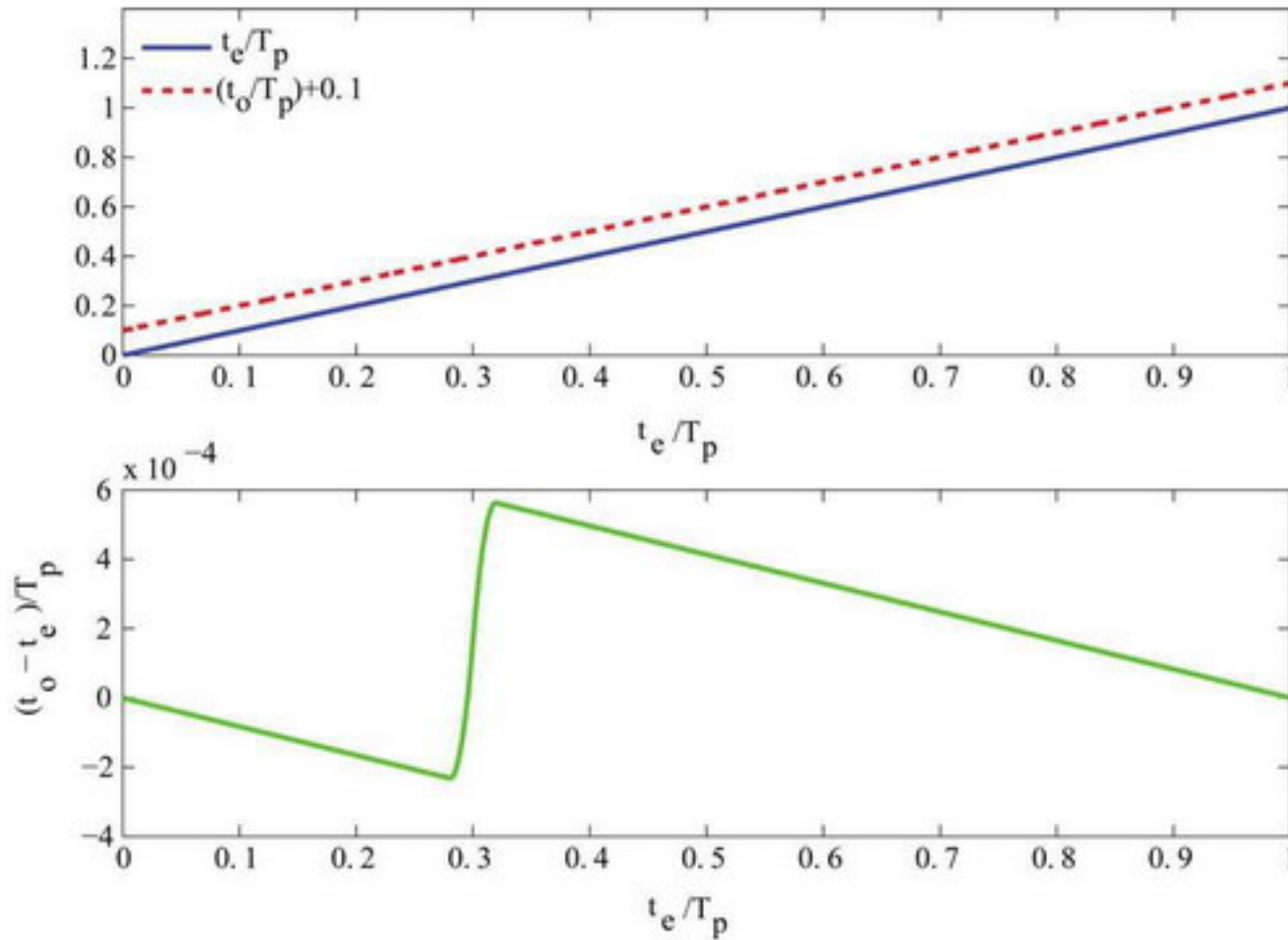


FIG. 3. (Color online) Comparison of normalized, observation time interval  $(t_o/T_p)$  in Eq. (28) with parameter  $\Gamma_p = 4 \times 10^{-4}$ , and normalized, emission time interval  $(t_e/T_p)$  for the photosphere motion in Fig. 2.

which results in

$$\log_{10} \left[ \frac{B_o(t_o)}{B^*} \right] = \left[ \frac{1}{\ln(10)} \left( \frac{d_{e,o}}{c^2} \right) \right] a_p(t_e) \propto a_p(t_e). \quad (32)$$

Thus, the variation in the observed brightness  $\log_{10}[B_o(t_o)]$  (i.e., the light curve) reflects the photosphere acceleration when the photosphere motion is small and slowly varying. This similarity in the observed light curves is seen in Fig. 4, which compares the numerically computed brightness (e.g., photosphere motion) and the limit case in Eq. (32) for small, sufficiently slowly varying, photosphere motions.

Other types of photosphere acceleration (e.g., sinusoidal) are possible. The possible set of acceleration time patterns depends on the type of photosphere vibrations and the stellar dynamics. The difference between the shifted observation and emission times,  $t_o$  and  $t_e$ , increases with the parameter  $\Gamma_p$  in Eq. (28). Hence, the brightness variation can change substantially from the limit case, in Eq. (32). To illustrate, the parameter  $\Gamma_p$  is increased from  $\Gamma_p = 4 \times 10^{-4}$  to  $\Gamma_p = 4$  and the resulting brightness variation (light curves) over a time period is shown in Fig. 5, which has similar patterns to typical light curves of binary stars.

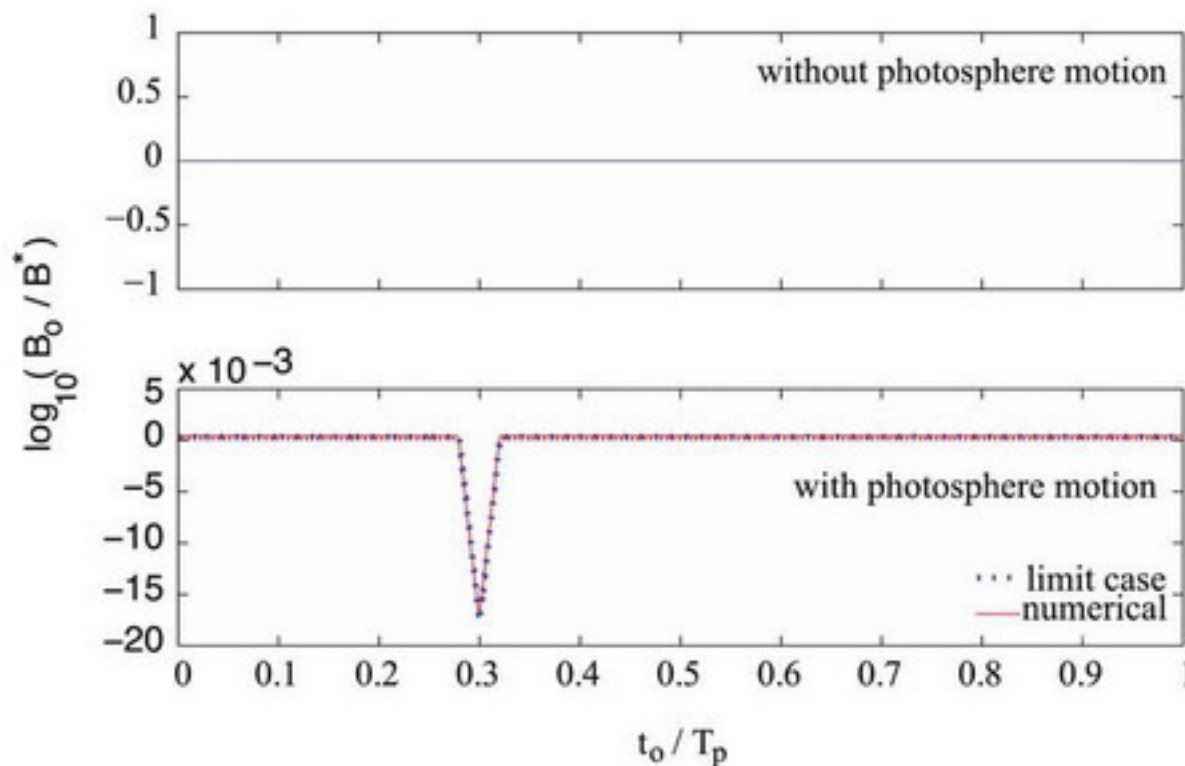


FIG. 4. (Color online) Comparison of observed brightness, without (top) and with (bottom) photosphere motion. The observed brightness  $B_o$  has a similar trend as the photosphere acceleration  $a_p$  (in Fig. 2) even though the energy-emission rate is uniform. Moreover, the observed brightness is close to the limit case for the example photosphere motion in Fig. 2.

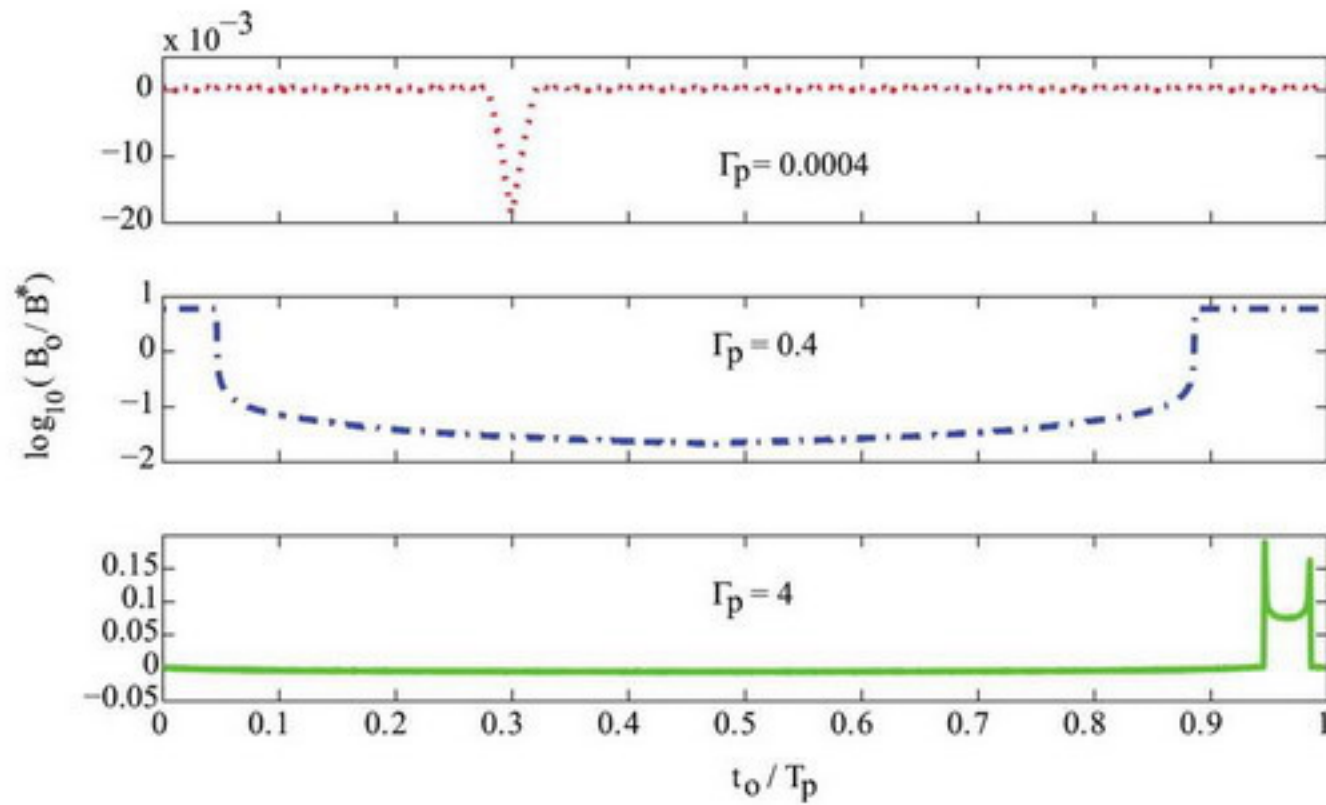


FIG. 5. (Color online) Observed light curves for different values of the parameter  $\Gamma_p$  in Eq. (28).

#### D. Photosphere vibration and apparent binaries

Different regions of the photosphere vibrations can have out-of-phase radial velocities, e.g.,  $v_p$  and  $-v_p$  at the same time instant (say,  $t_{e,i}$ ). Note that such out-of-phase, radial velocities readily occur in flexural vibrational modes of thin shells—these flexural modes can have lower associated resonance frequencies than the totally symmetric breathing (fundamental) mode of vibration.<sup>20</sup> Consequently, photons emitted from these regions (at  $t_{e,i}$ ) will arrive at the observer at different time instants, as in Eqs. (25) and (26),

$$t_{o,i+} = t_{e,i} + \frac{d_{e,o} - r_p(0)}{c + c\beta_p(0)}, \quad (33)$$

$$t_{o,i-} = t_{e,i} + \frac{d_{e,o} - r_p(t_e)}{c - c\beta_p(t_e)}. \quad (34)$$

This leads to a time shift ( $t_{o,i-} - t_{o,i+}$ ) between the observed light from these two regions as illustrated in Fig. 6. The photon energy observed from each region can be different, and depend on the relative size (and energy-emission rate) of each region. Since light from both regions are observed simultaneously, the total light curve can show periodic changes that appear like binary systems. Moreover, two periodically varying, apparent velocities (red shifts) will be observed as shown in Fig. 6 since the velocities (redshifts) of each region will be different. Such effects could account for large numbers of observed spectroscopic binaries with short time periods (associated with photosphere vibrations) even though visual binaries tend to have substantially larger, time periods.

Intermediate regions between the major vibrational regions will distort the simple addition of the two shifted light curves in Fig. 6. This distortion will depend on the relative size (and energy-emission rate) of the major and intermediate regions. Moreover, the spread of redshifts could lead a time-varying thickening of the spectrum bands rather

than generating discrete values in the spectrum as in the above example. Higher-order vibrational modes can lead to multiple regions with substantially different phase. This can lead to more than two, shifted, light curves being observed simultaneously resulting in apparent multiple-star systems. These issues are not considered in this study, for simplicity. Nevertheless, the proposed Ritz-type model indicates that photosphere vibration can lead to observations that appear to be from binary or multistar systems.

#### IV. QUASAR DISTANCE AND TIME DILATION

In this section, the cosmological expansion effect (that was neglected in Sec. III, which studied nearby stars) is included when computing the time between emission and observation. The results are used to generate potential explanations for the apparent lack of time dilation in quasar light curves,<sup>13</sup> as well as the observational links between quasars and nearby-galaxies.<sup>14,15</sup>

##### A. Time-dilation expression

Consider the case when a photon is emitted by a moving photosphere, which is considered to be spherical about the emitter  $e$  as in Fig. 1. Let two photons be emitted at time instants  $t_{e1} = t_{e,i}$  and  $t_{e2} = t_{e,i} + t_e$ , which reach the observer  $o$  at time instants  $t_{o1}$  and  $t_{o2}$ , respectively. Since the center of the emitter is moving with speed  $(c - c_o)$  away from the observer due to cosmological expansion, in Eq. (4), during the emission time-interval  $t_e = t_{e2} - t_{e1}$ , the initial distance  $d(e_1, o_1)$  (between the centers of the emitter and the observer) has increased by  $V_{e,o}t_e = (c - c_o)t_e$  to the final distance  $d(e_2, o_2)$ , as illustrated in Fig. 7. Moreover, let the radius of the photosphere, at the two emission instants, be  $r_{p1} = r_p(t_{e1})$  and  $r_{p2} = r_p(t_{e2})$  with expansion rates  $c\beta_{p1} = c\beta_p(t_{e1})$  and  $c\beta_{p2} = c\beta_p(t_{e2})$ . Consequently, the photons emitted at time instants  $t_{e1}$  and  $t_{e2}$  move toward the observer at speeds  $v_1 = c_o + c\beta_1$  and  $v_2 = c_o + c\beta_2$ ,

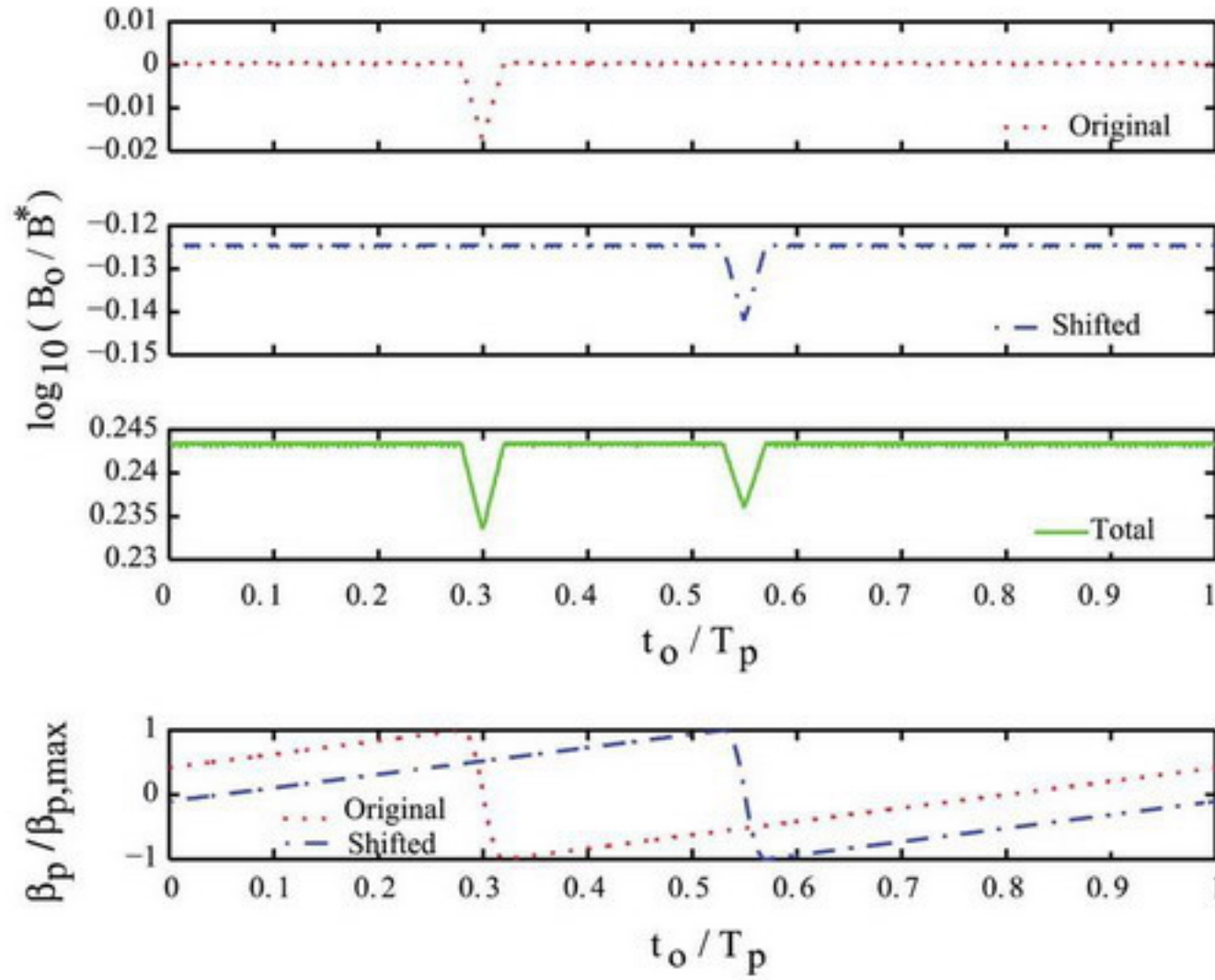


FIG. 6. (Color online) Apparent binaries: (Top three plots) Observed brightness  $B_o$ . The observed time shift (from the original brightness plot in Fig. 4) is a quarter of the time period ( $T_p/4$ ) and the brightness of the shifted light curve is 3/4th of the original brightness. (Bottom plot) Observed photosphere speed ( $\beta_p/\beta_{p,max}$ ), where at the same (shifted observer) time instant  $t_o$ , two values of photosphere speed can be observed, which could appear similar to observations from a spectroscopic binary.

$$t_{o1} = t_{e1} + \frac{d(e_1, o_1) - r_{p1}}{c_o + c\beta_{p1}}, \quad (35)$$

$$t_{o2} = t_{e2} + \frac{d(e_2, o_2) + (c - c_o)t_e - r_{p2}}{c_o + c\beta_{p2}}. \quad (36)$$

Therefore, the time interval  $t_o$  between observations is (provided  $t_{o2} \geq t_{o1}$ )

$$\begin{aligned} t_o(t_e) &= t_{o2} - t_{o1} = t_e + \frac{d(e_1, o_1) + (c - c_o)t_e - r_{p2}}{c_o + c\beta_{p2}} \\ &\quad - \frac{d(e_1, o_1) - r_{p1}}{c_o + c\beta_{p1}}, \\ &= t_e \left[ 1 + \frac{(1 - c_o/c)}{(c_o/c + \beta_{p2})} \right] \\ &\quad - \left[ \frac{(\beta_{p2} - \beta_{p1}) \frac{d(e_1, o_1)}{c}}{(c_o/c + \beta_{p2})(c_o/c + \beta_{p1})} \right] \\ &\quad - \left[ \frac{(r_{p2}/c)}{(c_o/c + \beta_{p2})} - \frac{(r_{p1}/c)}{(c_o/c + \beta_{p1})} \right], \end{aligned} \quad (37)$$

which is similar to the expression in Eq. (27) for the case without cosmological expansion in Sec. III. The main difference between the two cases is in the first term, where an additional expression is present in Eq. (37) for the case with cosmological expansion. This additional term tends to zero when the cosmological redshift is small. Moreover, the observation interval  $t_o$  becomes substantially large as the

distance  $d(e_1, o_1)$  increases and the second term in Eq. (37) dominates the expression—even for a small change in the photosphere speed over the emission interval  $t_e$ . Large changes in the observation time interval can lead to

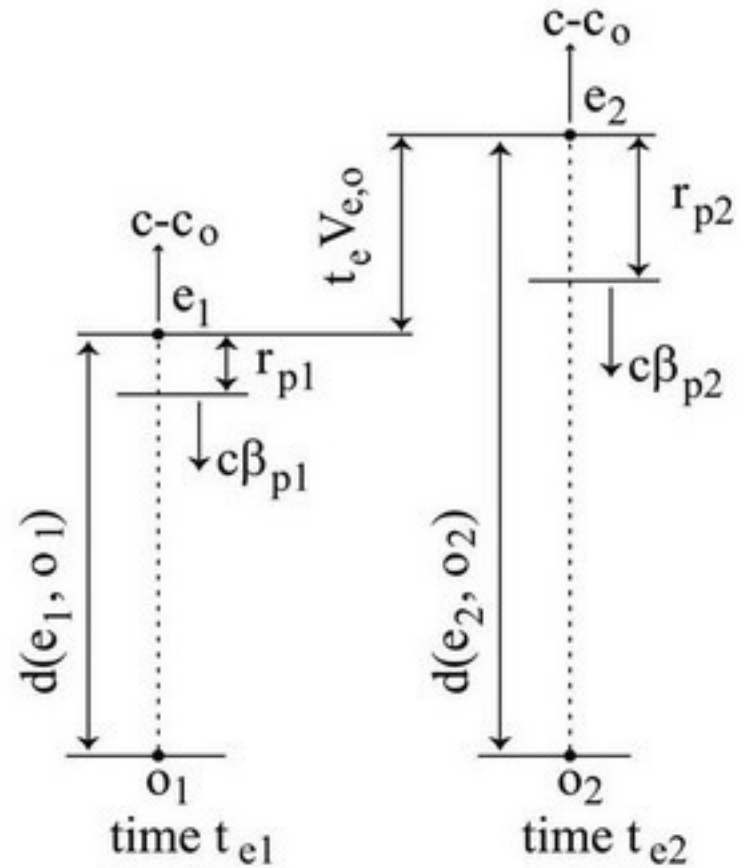


FIG. 7. Light emitted from point  $e_1$  at time  $t_{e1}$  and from point  $e_2$  at time  $t_{e2} = t_{e1} + t_e$ . The position of the photosphere is  $r_{p1}$  and  $r_{p2}$  away from the emitter center (toward the observer) and the corresponding speeds of the photosphere are  $c\beta_{p1}$  and  $c\beta_{p2}$ . The relative speed  $V_{e,o}$  of the emitter  $e$  with respect to the observer  $o$  is  $(c - c_o)$ . In the reference frame of the observer  $I_o$ , in which the location of the observer is fixed, i.e.,  $o_1 = o_2$ .



redistributions of the light curve over the time period that can appear to be chaotic—similar advent of chaos has been studied in stellar dynamics, e.g., Ref. 21.

The relation between emission and observation intervals in Eq. (37) can be rewritten in terms of the cosmological redshift  $z$  using Eqs. (2), (9), (10), and (19) as

$$t_o(t_e) = t_e \left[ 1 + \frac{z}{(1 + \beta_{p2}(1 + z))} \right] - \left[ \frac{(\beta_{p2} - \beta_{p1})H^{-1}z}{(1 + \beta_{p1}(1 + z))(1 + \beta_{p2}(1 + z))} \right] - \left[ \frac{(r_{p2}/c)(1 + z)}{(1 + \beta_{p2}(1 + z))} - \frac{(r_{p1}/c)(1 + z)}{(1 + \beta_{p1}(1 + z))} \right]. \quad (38)$$

Note that the second term in Eq. (38) relates to the speed variations in the photosphere and the third term includes radii variations of the photosphere. The time dilation expression in Eq. (38) collapses to the expression  $t_o = (1 + z)t_e$  in Eq. (12) if the photosphere radius is not varying, i.e.,  $r_{p1} = r_{p2}$  and the speeds  $\beta_{p1}$  and  $\beta_{p2}$  are zero.

**B. Periodic pulse emissions**

Consider the case, when emissions are periodic pulses occurring when the photosphere has a specific speed  $c\beta_p$  and radius  $r_p$ . This could occur, for example, if the emission accompanies (periodic) collapses of the stellar system. Then, the relation in Eq. (38) between the emission and observation time intervals  $t_e$  and  $t_o$  reduces to

$$t_o = t_e \left[ 1 + \frac{z}{1 + \beta_p(1 + z)} \right], \quad (39)$$

where  $z$  is the cosmological redshift. Note that this expression for time dilation in Eq. (39) is similar to that in Eq. (12)—the difference is the additional effect of the photosphere speed (i.e.,  $\beta_p$ ) at the instant the pulses are emitted in Eq. (39).

**C. Spectroscopic versus cosmological redshift**

The time dilation can be expressed in terms of the measured spectroscopic redshift (that includes the effect of photosphere motion, i.e.,  $\beta_p \neq 0$ ) rather than the cosmological redshift  $z$  in Eq. (9) due to cosmological expansion alone without photosphere motion, i.e.,  $\beta_p = 0$ . Toward this, the cosmological redshift  $z$  is compared with the spectroscopic redshift  $z_s$  that is given by, similar to Eq. (9),

$$1 + z_s = \frac{c}{c_o + c\beta_p}, \quad (40)$$

provided

$$c_o + c\beta_p > 0, \quad (41)$$

which is important to ensure that emitted photons have a positive speed toward (and reach) the observer. The

spectroscopic redshift  $z_s$  can be related to the cosmological redshift  $z$  by using Eq. (9) as

$$1 + z_s = \frac{1}{\frac{1}{1 + z} + \beta_p}, \quad (42)$$

which can be rewritten as

$$(1 + z) = \frac{(1 + z_s)}{1 - \beta_p(1 + z_s)}. \quad (43)$$

The spectroscopic redshift  $z_s$  approaches infinity as the cosmological redshift  $z$  reaches a critical value  $z_c$  and the net speed of photons tends to zero. In particular, for photons to reach an observer, Eq. (41) should be satisfied, which also implies that the denominator of Eq. (42) should remain positive, i.e.,

$$(1 + z)\beta_p > -1, \quad (44)$$

yielding an expression for the critical redshift  $z_c$  for a collapsing photosphere ( $\beta_p < 0$ ) as

$$z < z_c = \frac{1}{|\beta_p|} - 1 \quad \text{if } \beta_p < 0. \quad (45)$$

**D. Distance to quasars**

When the photosphere speed is small,  $\beta_p \rightarrow 0$ , the spectroscopic redshift  $z_s$  approaches the cosmological redshift  $z$  as seen in Eq. (42). However, the spectroscopic redshift  $z_s$  can be large compared to the cosmological redshift  $z$  when  $z \rightarrow z_c$  as illustrated in Fig. 8. Therefore, the Hubble law in Eq. (18) would indicate that an object is substantially further away if the spectroscopic redshift  $z_s$  is used instead of the cosmological redshift  $z$ . In other words, high spectroscopically redshifted quasars might be nearer than

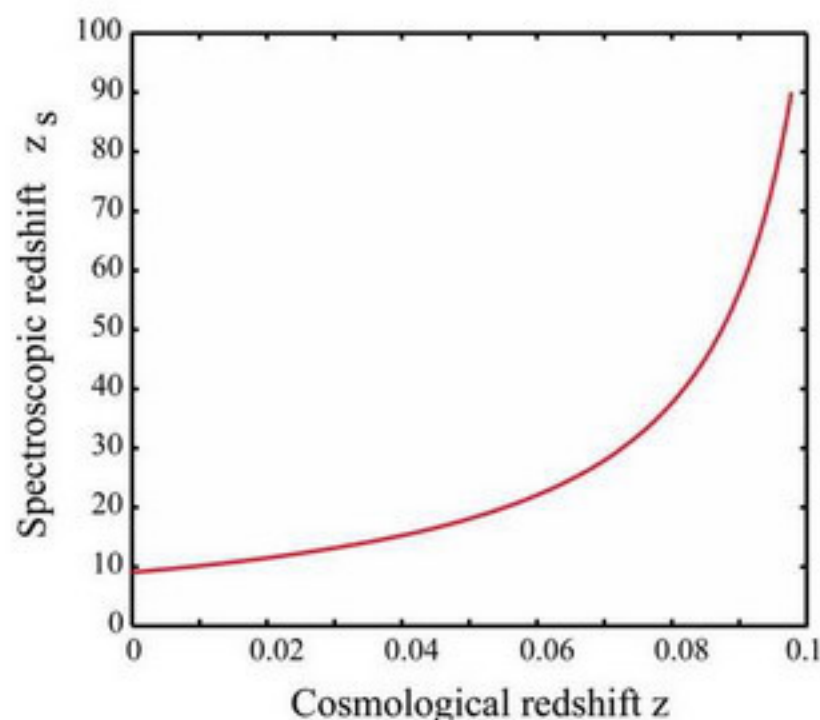


FIG. 8. (Color online) Spectroscopic  $z_s$  versus cosmological  $z$  redshift with an example photosphere speed of  $v_p = -0.9c$  ( $\beta_p = -0.9$ ). This can lead to erroneous distance (and age) estimates from the Hubble law in Eq. (18) if the spectroscopic redshift  $z_s$  is used instead of the cosmological redshift  $z$ .

previously thought, which could explain, e.g., the quasar redshift-distance anomalies such as potential links between high-(spectroscopic)-redshift quasars and lower-redshift galaxies.<sup>14,15</sup> If the distance is not as large as thought, then it is possible that ejecta observations at superluminal speeds<sup>16</sup> might be traveling at much smaller speeds—although the model presented here does not preclude superluminal speeds.

### E. Quasar time dilation

The time-dilation relation in Eq. (39) between the emission and observation time intervals  $t_e$  and  $t_o$  can be expressed in terms of the spectroscopic redshift  $z_s$  as

$$\frac{t_o}{t_e} = 1 + \frac{\left[ \frac{z_s + \beta_p(1+z_s)}{1 - \beta_p(1+z_s)} \right]}{\left[ 1 + \frac{\beta_p(1+z_s)}{1 - \beta_p(1+z_s)} \right]} = (1+z_s)(1+\beta_p). \quad (46)$$

Therefore, the time dilation is different by  $(1+\beta_p)$  when compared to the standard expression obtained by replacing the cosmological redshift  $z$  in Eq. (12) by the spectroscopic redshift  $z_s$ , i.e.,

$$\frac{t_o}{t_e} = (1+z_s). \quad (47)$$

Thus, for a collapsing system, with large negative photosphere speeds, e.g.,  $\beta_p = -0.9$ , the actual time dilation from Eq. (46) can be substantially lower than the time dilation predicted by the standard expression in Eq. (47) as seen in Fig. 9. This could explain recent analysis that appears to indicate that quasars do not show anticipated time dilation effects even with substantial spectroscopic redshifts.<sup>13</sup>

Emissions from quasar can have time dependency, which is different from the periodic pulses considered in the above analysis. Additionally, variations in the photosphere speeds can lead to more complex light curves (being observed) due to variations in the time dilation as predicted

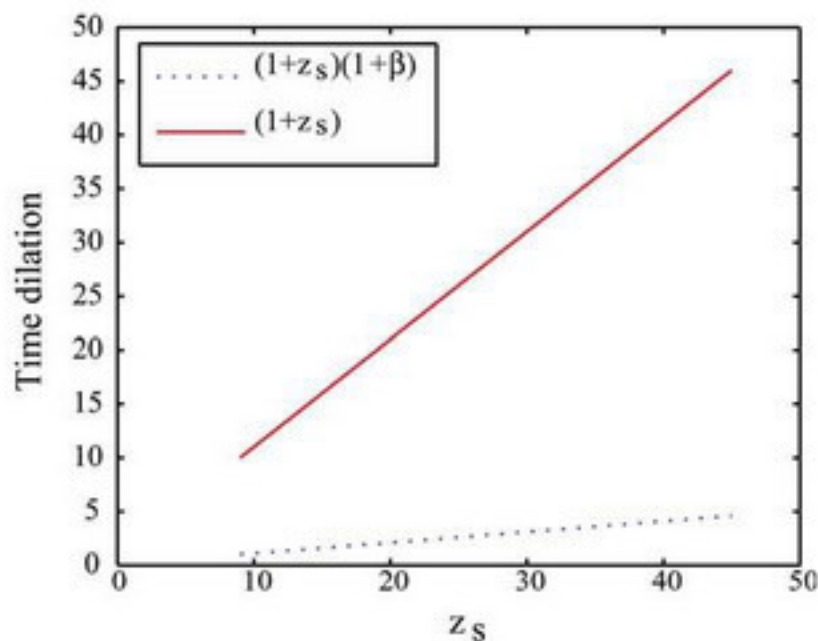


FIG. 9. (Color online) Comparison of time-dilation predictions. Time dilation predictions (solid line) based on the spectroscopic redshift  $z_s$  as in Eq. (47) can be substantially higher than the time-dilation (dotted line) based on the proposed model in Eq. (46). The difference depends on the photosphere speed, which is  $v_p = -0.9c$  ( $\beta_p = -0.9$ ) for this plot.

by Eq. (38)—these are not considered here for simplicity. Nevertheless, the above analysis shows the potential of the proposed model to account for the absence of time dilation in quasar light curves.<sup>13</sup> Moreover, the difference between the spectroscopic redshift  $z_s$  and cosmological redshift  $z$  in the proposed model could explain the anomaly between smaller apparent distance (due to potential links to low redshift galaxies as well as the presence of superluminal ejecta) and large quasar (spectroscopic) redshifts.<sup>14–16</sup>

### V. CONSISTENCY WITH FARTHER-DIMMER EFFECT

The effect of photosphere-speed variation is evaluated as a potential explanation of the farther-dimmer effect in SNe observations.<sup>17</sup>

#### A. Effect of speed variation on time dilation

Consider the case when the photosphere speed  $c\beta_p$  varies slightly over the time interval  $t_e$ . Then, the observed time dilation expression in Eq. (38) can be simplified by setting  $\beta_{p1} = \beta_p$ ,  $\beta_{p2} = \beta_p + \alpha_p t_e$ , and  $\beta_{p2} - \beta_{p1} = \alpha_p t_e$ , where  $\alpha_p$  is the normalized photosphere acceleration. The overall photosphere changes are assumed to be small, i.e.,

$$1 + \beta_{p2}(1+z) \approx 1 + \beta_{p1}(1+z) \approx 1 + \beta_p(1+z), \quad (48)$$

and  $r_{p2} \approx r_{p1} + c\beta_p t_e$ , to obtain

$$t_o(t_e) \approx t_e(1+z) \frac{\left[ 1 - \left( \frac{z}{1+z} \right) \frac{\alpha_p H^{-1}}{1 + \beta_p(1+z)} \right]}{[1 + \beta_p(1+z)]}. \quad (49)$$

#### B. Modified Hubble law

Substituting the modified time-dilation expression in Eq. (49) into the derivation of the observed brightness, i.e., in Eq. (15), the observed brightness of SNe  $B_{SN}$ , i.e.,  $B_o$  in Eq. (16), can be rewritten as

$$\begin{aligned} B_{SN} &= \frac{L_e}{4\pi \left( \frac{Rc}{V} \right)^2 \left( \frac{z^2}{1+z} \right)} \\ &\times \left[ \frac{1 + \beta_p(1+z)}{(1+z) \left[ 1 - \left( \frac{z}{1+z} \right) \frac{\alpha_p H^{-1}}{1 + \beta_p(1+z)} \right]} \right] \\ &= \frac{L_e}{4\pi (d_{L,SN})^2}, \end{aligned} \quad (50)$$

with the following modified Hubble law in Eq. (18) for SNe observations due to changes in the photosphere speed (represented by the term  $\alpha_p$ ):

$$d_{L,SN} = \frac{Rc}{V} z \sqrt{\frac{1 - \left( \frac{z}{1+z} \right) \frac{\alpha_p H^{-1}}{1 + \beta_p(1+z)}}{1 + \beta_p(1+z)}}, \quad (51)$$

where  $d_{L,SN}$  is the luminosity distance for supernova observations. For small photosphere speeds (e.g.,  $\beta_p = 0.033$  (Ref. 22)), the above expression can be simplified, further, to

$$d_{L,SN} \approx \frac{Rc}{V} z \sqrt{1 - \left(\frac{z}{1+z}\right) \alpha_p H^{-1}}. \quad (52)$$

### C. Farther dimmer with decelerating photosphere

If the photosphere acceleration is negative ( $\alpha_p < 0$ ), then the time dilation in Eq. (49) is increased—this results in an increase in the luminosity distance, e.g., in Eq. (52). This offers a potential explanation of the farther–dimmer effect,<sup>17</sup> as discussed below.

Based on the Hubble law in Eq. (18), the variation of the observed (normalized) blue-band maximum  $\mu_B$  from supernova (Type Ia) observations as a function of the host-galaxy redshift can be expressed in terms of the luminosity distance  $d_L$  (or, rather, the redshift  $z$ ) as

$$\mu_B = K_B + 5 \log_{10} \left[ \frac{V}{Rc} d_L \right] = K_B + 5 \log_{10}[z], \quad (53)$$

where  $K_B$  is a constant. This does not match the observed data from the Supernova Cosmology Project (SCP)<sup>23</sup> as shown in Fig. 10—leading to the possibility of an accelerating universe.

With the proposed model, the variation of observed maximum light can be expressed in terms of the luminosity distance  $d_{L,SN}$  from Eq. (51)

$$\begin{aligned} \mu_B &= K_B + 5 \log_{10} \left[ \frac{V}{Rc} d_{L,SN} \right] \\ &= K_B + 5 \log_{10} \left[ z \left[ \frac{1 - \left(\frac{z}{1+z}\right) \frac{\alpha_p H^{-1}}{1 + \beta_p(1+z)}}{1 + \beta_p(1+z)} \right]^{1/2} \right], \end{aligned} \quad (54)$$

which reduces to, at small photosphere speeds,  $\beta_p \rightarrow 0$ ,

$$\mu_B(z) = K_B + 5 \log_{10} \left[ z \left( 1 - \frac{z}{1+z} \alpha_p H^{-1} \right)^{1/2} \right]. \quad (55)$$

The parameters  $K_B = 43.13$  and  $\alpha_p H^{-1} = -3.18$  were estimated by minimizing the least square error between the model's prediction from Eq. (54) and the observed data ( $N_{SCP} = 307$  pairs of blue-band maximum  $\mu_{B,i}$  and redshift  $z_i$  with index  $i = 1, \dots, N_{SCP}$ ) from the SCP<sup>23</sup> at an example photosphere speed of  $\beta_p = 0.033$ .<sup>22</sup> The residual error  $E_{SCP,i}$  between the observed blue-band maximum  $\mu_{B,i}$  and the predictions  $\mu_B(z_i)$  from Eq. (55) at the observed redshift  $z_i$  is defined as

$$E_{SCP,i} = \mu_{B,i} - \mu_B(z_i). \quad (56)$$

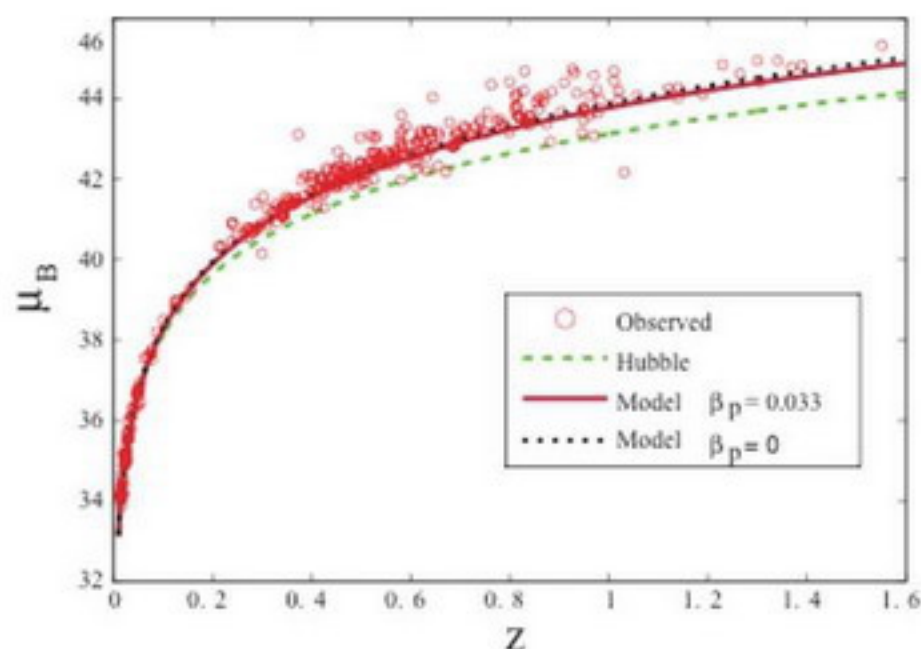


FIG. 10. (Color online) Hubble diagram (solid line) with the photosphere motion model in Eq. (51) with  $\beta_p = 0.033$  captures the farther–dimmer relationship in supernova observations (circles) when compared to the predictions in Eq. (53) (dashed line) from the standard Hubble law. The prediction from the simplified model in Eq. (55) with  $\beta_p = 0$  is shown for comparison—the simplified prediction varies by less than 0.1 from the model with the nominal photosphere speed of  $\beta_p = 0.033$ . The circles represent 307 SNe data (that are not outliers) from the SCP [23].

The error norm  $E_{SCP}$ ,

$$E_{SCP} = \sqrt{\sum_{i=1}^{N_{SCP}} [E_{SCP,i}]^2}, \quad (57)$$

over all observations is plotted for different values of  $K_B$  and  $\alpha_p H^{-1}$  in Fig. 11. The central ellipse in Fig. 11 represents the minimum of the error norm  $E_{SCP}$ , which led to parameter estimates of  $K_B = 43.13$  and  $\alpha_p H^{-1} = -3.18$ . The resulting residual error  $E_{SCP,i}$  as in Eq. (56), shown in Fig. 11, has low correlation with the redshift—the correlation  $R_{residual}$  between the residual error  $E_{SCP,i}$  and the redshift  $z_i$  is  $R_{residual} = -0.0173$  with a 95% confidence interval  $(-0.1290, 0.0948)$ . Thus, the proposed model can account for the farther–dimmer relationship in supernova observations,<sup>17</sup> without an accelerating universe.

## VI. TEMPORAL AND SPATIAL DISTORTIONS

Variations in the speed of photons can cause temporal and spatial distortions in astronomical observations. Such distortions are discussed below for two cases: (i) temporal distortion in SNe light curves and (ii) spatial distortion of mass distribution in galaxies.

### A. Temporal distortion in SNe light curves

An aspect of the proposed model is the potential for apparent time reversal. In Sec. V C, deceleration of the photosphere was shown to result in different photon speeds, which in turn, causes additional time dilation. A similar effect is possible, even with an accelerating photosphere due to apparent time reversal. In particular, a photon emitted at time  $t_{e2}$  could be observed earlier than a photon emitted earlier at time  $t_{e1} < t_{e2}$ . Let the corresponding observation times be  $t_{o1}$  (photon emitted earlier) and  $t_{o2}$  with  $t_{o1} > t_{o2}$  as in

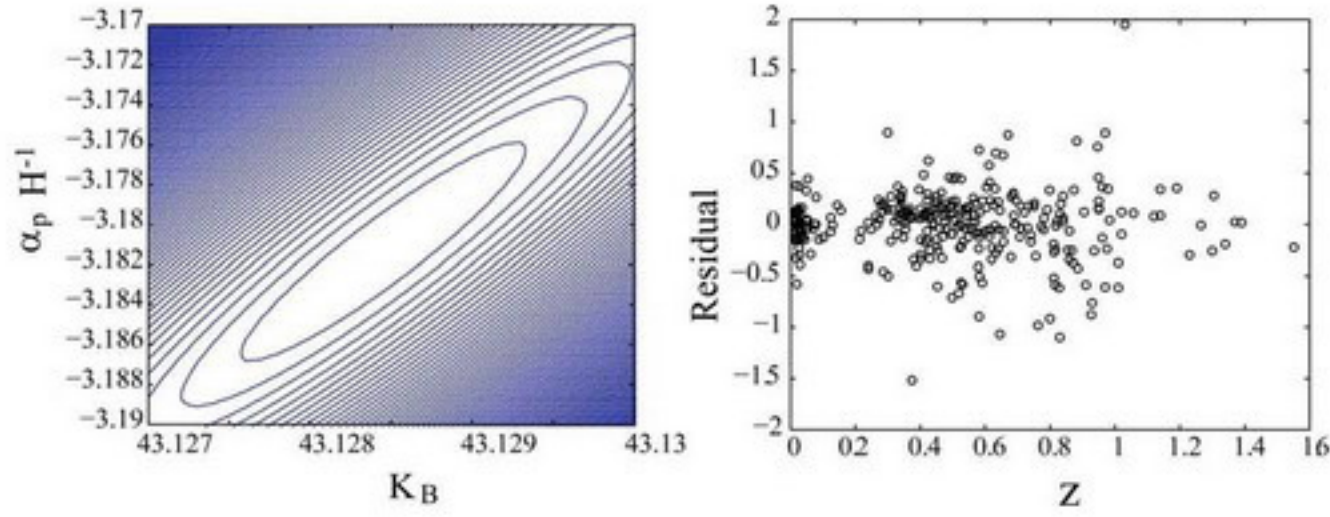


FIG. 11. (Color online) Left error norm  $E_{SCP}$  from Eq. (57) for different model parameters  $K_B$  and  $\alpha_p H^{-1}$  from Eq. (55). The central ellipse in Fig. 11 represents the minimum of the error norm  $E_{SCP}$ , which led to parameter estimates of  $K_B = 43.13$  and  $\alpha_p H^{-1} = -3.18$ . Right: the resulting residual error  $E_{SCP,i}$  in Eq. (56) has low correlation,  $R_{\text{residual}} = -0.0173$ , with redshift  $z_i$ .

Eq. (36). In this case, the time interval  $t_o$  between observations is, reversing the observation time intervals  $t_{o1}$  and  $t_{o2}$  in Eq. (37),

$$\begin{aligned}
 t_o(t_e) &= t_{o1} - t_{o2} \\
 &= -t_e - \frac{d(e_1, o_1) + (c - c_o)t_e - r_{p2}}{c_o + c\beta_{p2}} \\
 &\quad + \frac{d(e_1, o_1) - r_{p1}}{c_o + c\beta_{p1}}, \\
 &= -t_e \left[ 1 + \frac{(1 - c_o/c)}{(c_o/c + \beta_{p2})} \right] \\
 &\quad + \left[ \frac{(\beta_{p2} - \beta_{p1}) \frac{d(e_1, o_1)}{c}}{(c_o/c + \beta_{p2})(c_o/c + \beta_{p1})} \right] \\
 &\quad + \left[ \frac{(r_{p2}/c)}{(c_o/c + \beta_{p2})} - \frac{(r_{p1}/c)}{(c_o/c + \beta_{p1})} \right], \quad (58)
 \end{aligned}$$

which can be positive  $t_o(t_e) > 0$  with an accelerating photosphere provided the second term in Eq. (58) is positive and dominates the other two terms in Eq. (58), i.e., the speed of the later photon  $\beta_{p2}$  is greater than the speed  $\beta_{p1}$  of the earlier photon, there is sufficient travel distance  $d(e_1, o_1)$  for the later photon to overtake the earlier photon, and there is no interference between the photons.

The apparent time reversal can lead to SNe light with the highest photosphere speeds to be observed first, with a continuous decrease in the observed photosphere speed over time—such a reduction in photosphere speed is present in current SNe observations.<sup>22</sup> Moreover, if the photosphere was accelerating during the explosion, the apparent time reversal can lead to apparent reversal in the direction of the observed shock waves, i.e., they might appear to propagate backward toward the supernova center. Further study is needed to evaluate if the reverse shock waves, seen in some of the SNe remnants,<sup>24</sup> could be caused by such an effect. Finally, apparent time reversal also implies that light from the host galaxy (which might not have the large photosphere velocities as SNe) travel at a slower speed when compared to the associated SNe. Consequently, the light from the host galaxy (observed at the same time as the SNe) can be much

older than the SNe. This difference in age (between SNe and their host galaxies) could account for the difficulty in identifying SNe progenitors.<sup>25</sup>

## B. Spatial distortion of astronomical structures

Variations in the arrival speed (of photons) can lead to distortions in the observed mass distribution of astronomical structures such as galaxies. If not accounted for, apparent spatial distortions can raise challenges in modeling the dynamics of observed physical phenomena such as the rotational dynamics of galaxies, which are dependent on the mass distribution. To illustrate, consider an astronomical structure, illustrated by the disk in Fig. 12 that is rotating about an axis  $EO$  that passes through the disk center  $E$  and is perpendicular to the disk. Consider light emitted from a point  $A$  on the disk, which is at a distance  $r = d(E, A)$  from the disk center  $E$  with tangential speed  $V_g(r) = c\beta_g(r)$ . Let the observer be at location  $O$ , where the distance  $d = d(E, O)$  is large compared to the radius  $r$ , which allows the approximation  $d(A, O) \approx d$ . Then, the time  $t(r)$  for a photon to travel to the observer at  $O$  is given by

$$\begin{aligned}
 t(r) &= \frac{d(A, O)}{c_g} \\
 &= \frac{d(A, O)}{c\sqrt{1 - [\beta_g(r)]^2}} \approx \frac{d}{c\sqrt{1 - [\beta_g(r)]^2}}, \quad (59)
 \end{aligned}$$

where cosmological expansion effects are neglected in the relative speed of light  $c_g$ . The difference between the travel times for emission from the center  $E$  and the emission from the point  $A$  is given by

$$\begin{aligned}
 \Delta t(r) &= t(r) - t(0) \\
 &= \frac{d}{c\sqrt{1 - [\beta_g(r)]^2}} - \frac{d}{c} \\
 &= \frac{d}{c} \left[ \frac{1 - \sqrt{1 - [\beta_g(r)]^2}}{\sqrt{1 - [\beta_g(r)]^2}} \right]. \quad (60)
 \end{aligned}$$

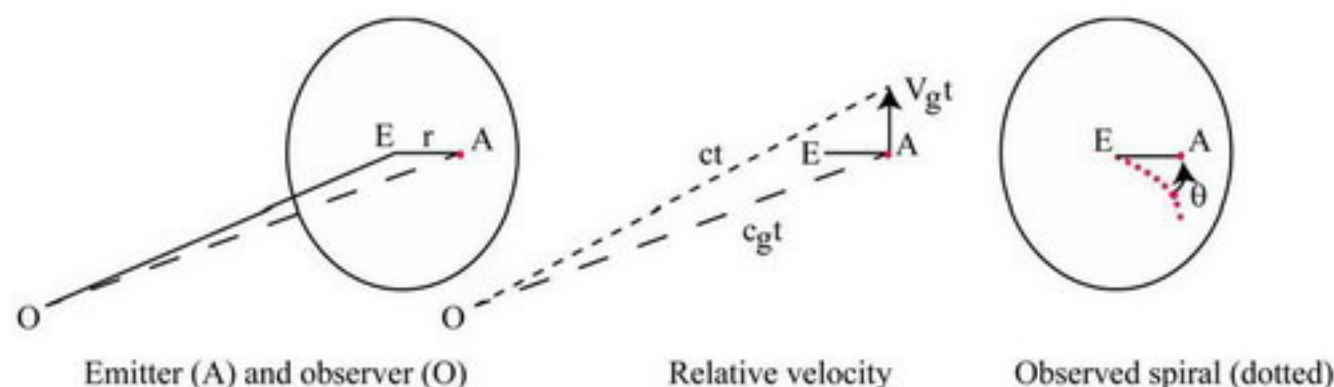


FIG. 12. (Color online) Spiral spatial distortion. Travel times for emissions from  $A$  on a disk at radial distance  $r$  from the disk center  $E$  can vary with the radial distance  $d$  as in Eq. (59)—this can lead to the straight segment  $EA$  appearing as a spiral when observed at  $O$ .

The above expression can be simplified, for small tangential speeds ( $\beta_g(r) \rightarrow 0$ ), as

$$\Delta t(r) = \frac{d}{c} \left[ \frac{[\beta_g(r)]^2}{2} \right]. \quad (61)$$

Note that the travel-time difference can be a function of the radial distance  $r$  from the center  $E$ , which can imply different angular rotations  $\theta(r)$  before an emission reaches the observer. In particular, if the angular rotation rate  $\omega(r)$  at each radius  $r$  is assumed to remain constant, then the time difference in Eq. (61) corresponds to a rotation angle  $\theta(r)$  given by

$$\begin{aligned} \theta(r) &= \omega(r) \Delta t(r) \\ &= \frac{c \beta_g(r)}{r} \left[ \frac{d [\beta_g(r)]^2}{c} \right] = \frac{d}{2r} [\beta_g(r)]^3. \end{aligned} \quad (62)$$

If, for example, the disk rotates as a rigid body at a constant angular rate  $\omega$ , then the tangential speed is given by

$$c \beta_g(r) = r \omega, \quad (63)$$

and the corresponding angular rotation  $\theta(r)$  during the travel time difference  $\Delta t(r)$  is given by

$$\theta(r) = \frac{d}{2r} \left[ \frac{r \omega}{c} \right]^3 \propto r^2. \quad (64)$$

Therefore, a straight line along the radius of the disk (e.g.,  $EA$ ) will appear to be a spiral due to increasing travel times for emissions along the length  $EA$ —as illustrated in Fig. 12. Such spatial distortion in observations needs to be corrected before studying the rotational dynamics since it can affect the mass distribution in the disk, and therefore, can affect the gravitational analysis. Similar spatial distortion can also occur along other directions such as the radial direction, e.g., in the presence of varying radial speeds if the disk collapses or expands, which will require further investigation.

## VII. THE GEOMETRY AXIOM

The spherical-shell geometry axiom allows for the total kinetic energy to be constant while satisfying momentum

conservation. The implications and potential relaxation of this axiom are discussed below.

### A. Peculiar velocities

The expansion rate is simplified to be a constant  $V$  (which maintains a constant kinetic energy) in the proposed cosmology model in contrast to a radial-distance-dependent speed variation as in Newtonian cosmology, e.g., Ref. 18. Nevertheless, a small expansion-rate variation across the thickness of the shell could be included in the proposed cosmology model. For example, components on the outside of the shell will have a net gravitational force toward the center of the shell—in contrast, there would be no such force on components on the inside of the shell. Therefore the speed  $V$  of the outer components in the shell (farther away from the center) is expected to reduce with respect to the inner components in the shell that are nearer to its center. Therefore, the inner components of the shell might appear to be attracted toward the outside components, which could explain observations such as peculiar velocities of galaxy clusters.<sup>26</sup>

### B. Anisotropy in cosmic microwave background radiation

The spherical shell geometry leads to variation between the radial and tangential directions. Moreover, even within the radial direction, there is anisotropy between the directions toward the center and away from the center. This should lead to anisotropy in observations such as the measured Hubble constant in different directions and in observations of the cosmic microwave background (CMB) radiation. Although some anisotropies have been reported in measurements of: (i) the Hubble constant (e.g., Ref. 27) and (ii) CMB radiation (e.g., quadrupole and octopole alignment<sup>28,29</sup>) further study is needed to check if such anisotropies are consistent with the proposed cosmology model.

### C. Other geometries

If anisotropies in cosmological observations are not observed, then the proposed cosmology model can be considered without the spherical-shell geometry axiom. In this case, the Hubble law cannot be derived as in Sec. II under the proposed model. Therefore, the expansion of the cosmos should be considered as an axiom, as in current cosmology models. Nevertheless, the other results of the model, such

as the explanation of apparent binaries in Sec. III, the farther-dimmer effect in Sec. V, and the temporal-and-spatial distortions in Sec. VI, would still be viable.

## VIII. CONCLUSIONS

This article developed a Ritz-type, VSL cosmology model, and evaluated its potential to match current cosmological observations. It was shown that the proposed model could explain some of the anomalies in current cosmological observations. Additional work is needed to evaluate potential variations in the Hubble constant and anisotropies in the CMB radiation due to differences between the radial and tangential directions in the spherical shell geometry to, both, test and potentially refine the model.

- <sup>1</sup>M. H. Shao, *Phys. Essays* **26**, 183 (2013).
- <sup>2</sup>D. Laskaroudis, *Phys. Essays* **26**, 452 (2013).
- <sup>3</sup>W. Petry, *Phys. Essays* **26**, 315 (2013).
- <sup>4</sup>L. Zaninetti, *Phys. Essays* **23**, 298 (2010).
- <sup>5</sup>R. B. Driscoll, *Phys. Essays* **23**, 584 (2010).
- <sup>6</sup>A. A. Martinez, *Phys. Perspect.* **6**, 4 (2004).
- <sup>7</sup>J. P. Petit, *Mod. Phys. Lett. A* **3**, 1733 (1988).
- <sup>8</sup>S. Devasia, *Z. Naturforsch.* **64a**, 327 (2009).
- <sup>9</sup>A. K. T. Assis and H. T. Silva, *Pramana J. Phys.* **55**, 393 (2000).
- <sup>10</sup>J. J. Turin and H. R. Crane, *Phys. Rev.* **52**, 610 (1937).
- <sup>11</sup>C. D. Ellis and W. A. Wooster, *Proc. R. Soc. London, Ser. A*, **117**, 109 (1927).
- <sup>12</sup>S. Blondin, T. M. Davis, K. Krisciunas, B. P. Schmidt, J. Sollerman, W. M. Wood Vasey, A. C. Becker, P. Challis, A. Clocchiatti, G. Damke, A. V. Filippenko, R. J. Foley, P. M. Garnavich, S. W. Jha, R. P. Kirshner, B. Leibundgut, W. Li, T. Matheson, G. Miknaitis, G. Narayan, G. Pignata, A. Rest, A. G. Riess, J. M. Silverman, R. C. Smith, J. Spyromilio, M. Stritzinger, C. W. Stubbs, N. B. Suntzeff, B. E. Tonry, J. L. Tucker, and A. Zenteno, *Astrophys. J.* **682**, 724 (2008).
- <sup>13</sup>M. R. S. Hawkins, *Mon. Not. R. Astron. Soc.* **405**, 1940 (2010).
- <sup>14</sup>H. C. Arp, *Quasars, Redshifts and Controversies* (Cambridge University Press, Cambridge, UK, 1987).
- <sup>15</sup>M. Lopez-Corredoira and C. M. Gutierrez, *Astron. Astrophys.* **461**, 393 (2007).
- <sup>16</sup>M. H. Cohen, K. I. Kellermann, D. B. Shaffer, R. P. Linfield, A. T. Moffet, J. D. Romney, G. A. Seielstad, I. I. K. Paulinytoth, E. Preuss, A. Witzel, R. T. Schilizzi, and B. J. Geldzahler, *Nature* **268**, 405 (1977).
- <sup>17</sup>A. G. Riess, A. V. Filippenko, W. Li, and B. P. Schmidt, *Astron. J.* **118**, 2668 (1999).
- <sup>18</sup>A. R. Thatcher, *Eur. J. Phys.* **3**, 202 (1982).
- <sup>19</sup>E. Hubble, *Astrophys. J.* **84**, 517 (1936).
- <sup>20</sup>T. A. Duffey, J. E. Pepin, A. N. Robertson, M. L. Steinzig, and K. Coleman, *J. Vib. Acoust.* **129**, 363 (2007).
- <sup>21</sup>V. Icke, A. Frank, and A. Heske, *Astron. Astrophys.* **258**, 341 (1992).
- <sup>22</sup>E. C. Pearce, S. A. Colgate, and A. G. Petschek, *Astrophys. J.* **325**, L33 (1988).
- <sup>23</sup>M. Kowalski, D. Rubin, G. Aldering, R. J. Agostinho, A. Amadon, R. Amanullah, C. Balland, K. Barbary, G. Blanc, P. J. Challis, A. Conley, N. V. Connolly, R. Covarrubias, K. S. Dawson, S. E. Deustua, R. Ellis, S. Fabbro, V. Fadeyev, X. Fan, B. Farris, G. Folatelli, B. L. Frye, G. Garavini, E. L. Gates, L. Germany, G. Goldhaber, B. Goldman, A. Goobar, D. E. Groom, J. Haissinski, D. Hardin, I. Hook, S. Kent, A. G. Kim, R. A. Knop, C. Lidman, E. V. Linder, J. Mendez, J. Meyers, G. J. Miller, M. Moniez, A. M. Mouro, H. Newberg, S. Nobili, P. E. Nugent, R. Pain, O. Perdereau, S. Perlmutter, M. M. Phillips, V. Prasad, R. Quimby, N. Regnault, J. Rich, E. P. Rubenstein, P. Ruiz-Lapuente, F. D. Santos, B. E. Schaefer, R. A. Schommer, R. C. Smith, A. M. Soderberg, A. L. Spadafora, L. G. Strolger, M. Strovink, N. B. Suntzeff, N. Suzuki, R. C. Thomas, N. A. Walton, L. Wang, W. M. Wood-Vasey, and J. L. Yun, *Astrophys. J.* **686**, 749 (2008).
- <sup>24</sup>K. Heng, R. McCray, S. A. Zhekov, P. M. Challis, R. A. Chevalier, A. P. S. Crotts, C. Fransson, P. Garnavich, R. P. Kirshner, S. S. Lawrence, P. Lundqvist, N. Panagia, C. S. J. Pun, N. Smith, J. Sollerman, and L. Wang, *Astrophys. J.* **644**, 959 (2006).
- <sup>25</sup>M. Livio and J. E. Pringle, *Astrophys. J. Lett.* **740**, 1 (2011).
- <sup>26</sup>A. Kashlinsky, F. Atrio-Barandela, D. Kocevski, and H. Ebeling, *Astrophys. J.* **686**, L49 (2008).
- <sup>27</sup>M. L. McClure and C. C. Dyer, *New Astron.* **12**, 533 (2007).
- <sup>28</sup>M. Tegmark, A. de Oliveira-Costa, and A. J. S. Hamilton, *Phys. Rev. D* **68**, (2003).
- <sup>29</sup>C. L. Bennett, R. S. Hill, G. Hinshaw, D. Larson, K. M. Smith, J. Dunkley, B. Gold, M. Halpern, N. Jarosik, A. Kogut, E. Komatsu, M. Limon, S. S. Meyer, M. R. Nolte, N. Odegard, L. Page, D. N. Spergel, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, *Astrophys. J. Suppl. Ser.* **192**, 1 (2011).